STRUCTURAL ANALYSIS – II

PART - A

UNIT - 1

ROLLING LOAD AND INFLUENCE LINES:
Rolling load analysis for simply supported beams for several point loads and UDL. Influence line diagram for reaction, SF and BM at a given section for the cases mentioned in above unit 1  
6 Hours

UNIT - 2

SLOPE DEFLECTION METHOD:
Introduction, Sign convention, Development of slope-deflection equations and Analysis of Beams and Orthogonal Rigid jointed plane frames (non-sway) with kinematic redundancy less than/equal to three. (Members to be axially rigid)  
8 Hours

UNIT - 3

MOMENT DISTRIBUTION METHOD:
Introduction, Definition of terms- Distribution factor, Carry over factor, Development of method and Analysis of beams and orthogonal rigid jointed plane frames (non-sway) with kinematic redundancy less than/equal to three. (Members to be axially rigid)  
8 Hours

UNIT - 4

SWAY ANALYSIS:
Analysis of rigid jointed plane frames (sway, members assumed to be axially rigid and kinematic redundancy £ 3) by slope deflection and moment distribution methods.  
4 Hours

PART - B

UNIT – 5

KANIS METHODS:
Introduction, Basic Concept, Analysis of Continuous beams and Analysis of rigid jointed non-sway plane frames.  
6 Hours

UNIT - 6

FLEXIBILITY MATRIX METHOD OF ANALYSIS:
Introduction, Development of flexibility matrix for plane truss element and axially rigid plane framed structural elements and Analysis of plane truss and axially rigid plane frames by flexibility method with static indeterminacy £3.  
7 Hours

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UNIT - 7

STIFFNESS MATRIX METHOD OF ANALYSIS:
Introduction, Development of stiffness matrix for plane truss element and axially rigid plane framed structural elements. And Analysis of plane truss and axially rigid plane frames by stiffness method with kinematic indeterminacy £3.

7 Hours

UNIT - 8

BASIC PRINCIPLES OF DYNAMICS:
Basic principles of Vibrations and causes, periodic and aperiodic motion, harmonic and non-harmonic motion. Period and frequency, Forced and Free Vibration, Damping and Equations of Single Degree of Freedom System with and without damping

6 Hours

REFERENCE BOOKS:
3. Structural Dynamics-by M.Mukhopadhyay,
5. **Basics of Structural Dynamics and Aseismic Design** By Damodhar Swamy and Kavita PHI Learning Private Limited
6. **Structural Analysis**- D.S. Prakash Rao,, A Unified Approach, University Press
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UNIT - 1
ROLLING LOAD AND INFLUENCE LINES

1 Introduction: Variable Loadings

So far in this course we have been dealing with structural systems subjected to a specific set of loads. However, it is not necessary that a structure is subjected to a single set of loads all of the time. For example, the single-lane bridge deck in Figure 1 may be subjected to one set of a loading at one point of time (Figure 1a) and the same structure may be subjected to another set of loading at a different point of time. It depends on the number of vehicles, position of vehicles and weight of vehicles. The variation of load in a structure results in variation in the response of the structure. For example, the internal forces change causing a variation in stresses that are generated in the structure. This becomes a critical consideration from design perspective, because a structure is designed primarily on the basis of the intensity and location of maximum stresses in the structure. Similarly, the location and magnitude of maximum deflection (which are also critical parameters for design) also become variables in case of variable loading. Thus, multiple sets of loading require multiple sets of analysis in order to obtain the critical response parameters.

![Figure 1 Loading condition on a bridge deck at different points of time](image)

*Influence lines* offer a quick and easy way of performing multiple analyses for a single structure. Response parameters such as *shear force* or *bending moment at a point* or *reaction at a support* for several load sets can be easily computed using influence lines.
For example, we can construct influence lines for (shear force at $B$) or (bending moment at) or (vertical reaction at support $D$) and each one will help us calculate the corresponding response parameter for different sets of loading on the beam $AD$ (Figure 2).

![Figure 2 Different response parameters for beam $AD$](image)

An influence line is a diagram which presents the variation of a certain response parameter due to the variation of the position of a unit concentrated load along the length of the structural member. Let us consider that a unit downward concentrated force is moving from point $A$ to point $B$ of the beam shown in Figure 3a. We can assume it to be a wheel of unit weight moving along the length of the beam. The magnitude of the vertical support reaction at $A$ will change depending on the location of this unit downward force. The influence line for (Figure 3b) gives us the value of for different locations of the moving unit load. From the ordinate of the influence line at $C$, we can say that when the unit load is at point $C$.

![Figure 3b Influence line of for beam $AB$](image)
Thus, an influence line can be defined as a curve, the ordinate to which at any abscissa gives the value of a particular response function due to a unit downward load acting at the point in the structure corresponding to the abscissa. The next section discusses how to construct influence lines using methods of equilibrium.

2 Construction of Influence Lines using Equilibrium Methods

The most basic method of obtaining influence line for a specific response parameter is to solve the static equilibrium equations for various locations of the unit load. The general procedure for constructing an influence line is described below.

1. Define the positive direction of the response parameter under consideration through a free body diagram of the whole system.
2. For a particular location of the unit load, solve for the equilibrium of the whole system and if required, as in the case of an internal force, also for a part of the member to obtain the response parameter for that location of the unit load. This gives the ordinate of the influence line at that particular location of the load.
3. Repeat this process for as many locations of the unit load as required to determine the shape of the influence line for the whole length of the member. It is often helpful if we can consider a generic location (or several locations) x of the unit load.
4. Joining ordinates for different locations of the unit load throughout the length of the member, we get the influence line for that particular response parameter. The following three examples show how to construct influence lines for a support reaction, a shear force and a bending moment for the simply supported beam AB.

Example 1 Draw the influence line for (vertical reaction at A) of beam AB in Fig.1

Solution:
Free body diagram of AB:

\[ \sum \mathcal{F}_y = 0 \Rightarrow R_A - 1 - R_B \]
\[ \sum M (\text{about B}) = 0 \Rightarrow R_A (L) = 1(L - x) \]
So the influence line of:

\[ R_A = 1 - \frac{x}{L} \]

**Example 2** Draw the influence line for (shear force at mid point) of beam \(AB\) in Fig.2.

Solution:

\[ \sum M(about \, \overline{BC}) = 0 \Rightarrow R_A = 1 - \frac{x}{L} \]

For \( x < \frac{L}{2} \)
\[ \sum F_p = 0 \Rightarrow V_C = 1 - R_A = \frac{x}{L} \]

For \( x > \frac{L}{2} \)

\[ \sum F_p = 0 \Rightarrow V_C = -R_A = \frac{x}{L} - 1 \]

So the influence line for \( V_C \):

\[
\begin{align*}
A & \quad \vdots \quad 0.5 \\
\quad & \quad \vdots \quad + \\
C & \quad \vdots \quad - \\
\quad & \quad \vdots \quad -0.5 \\
B & \quad \vdots \\
\end{align*}
\]

Example 3 Draw the influence line for (bending moment at \( D \)) for beam \( AB \) in Fig.3.

Solution:
\[ \sum M(\text{about} \ B) = 0 \Rightarrow R_A = 1 - \frac{x}{L} \]

For \[ x < \frac{2L}{3} \]

\[ \sum M(\text{about} \ D) = 0 \]
\[ \Rightarrow M_D = R_A \left( \frac{2L}{3} \right) - 1 \left( \frac{2L}{3} - x \right) - \left( 1 - \frac{x}{L} \right) \left( \frac{2L}{3} \right) - \frac{2L}{3} + x \]
\[ = - \frac{2x}{3} + x = \frac{x}{3} \]

For \[ x < \frac{2L}{3} \]

\[ \sum M(\text{about} \ D) = 0 \]
\[ \Rightarrow M_D = R_A \left( \frac{2L}{3} \right) - 1 \left( \frac{2L}{3} - x \right) - \left( 1 - \frac{x}{L} \right) \left( \frac{2L}{3} \right) - \frac{2L}{3} + x \]
\[ = - \frac{2x}{3} + x = \frac{x}{3} \]

For \[ x > \frac{2L}{3} \]

\[ \sum M(\text{about} \ D) = 0 \Rightarrow M_D = R_A \left( \frac{2L}{3} \right) - 2L - \frac{2x}{3} \]
Similarly, influence lines can be constructed for any other support reaction or internal force in the beam. However, one should note that equilibrium equations will not be sufficient to obtain influence lines in indeterminate structures, because we cannot solve for the internal forces/support reactions using only equilibrium conditions for such structures.

3 Use of Influence Lines

In this section, we will illustrate the use of influence lines through the influence lines that we have obtained in Section 2. Let us consider a general case of loading on the simply supported beam (Figure 4a) and use the influence lines to find out the response parameters for their loading. We can consider this loading as the sum of three different loading conditions, (A), (B) and (C) (Figure 4b), each containing only one externally applied force.

![Diagram of beam with loads](image)

Figure 4: Application of influence lines for a general loading: (a) all the loads, and (b) the general loading is divided into single force systems
For loading case (A), we can find out the response parameters using the three influence lines. Ordinate of an influence line gives the response for a unit load acting at a certain point. Therefore, we can multiply this ordinate by the magnitude of the force to get the response due to the real force at that point. Thus

\[ R_A = 5kN \times \text{(ordinate of the IL of } R_A \text{ at } x = 2m) = 5(1 - 2/6) = 3.33kN \]

\[ V_C = 5kN \times \text{(ordinate of the IL of } V_C \text{ at } x = 2m) = 5(2/6) = 1.67kN \]

\[ M_D = 5kN \times \text{(ordinate of the IL of } M_D \text{ at } x = 2m) = 5(2/3) = 3.33kNm \]

Similarly, for loading case (B):

\[ R_A = 4kN \times \text{(ordinate of the IL of } R_A \text{ at } x = 4m) = 4(1 - 4/6) = 1.33kN \]

\[ V_C = 4kN \times \text{(ordinate of the IL of } V_C \text{ at } x = 4m) = 4(4/6 - 1) = -1.33kN \]

\[ M_D = 4kN \times \text{(ordinate of the IL of } M_D \text{ at } x = 4m) = 4(2 \times 6/9) = 5.33kNm \]

And for case (C),

\[ R_A = -2kN \times \text{(ordinate of the IL of } R_A \text{ at } x = 5m) = -2(1 - 5/6) = -0.33kN \]

\[ V_C = -2kN \times \text{(ordinate of the IL of } V_C \text{ at } x = 5m) = -2(5/6 - 1) = 0.33kN \]

\[ M_D = -2kN \times \text{(ordinate of the IL of } M_D \text{ at } x = 5m) = -2(2 \times 6/3 - 2 \times 5/3) = -1.33kNm \]

By the theory of superposition, we can add forces for each individual case to find the response parameters for the original loading case (Figure 4a). Thus, the response parameters in the beam AB are:

\[ R_A = (3.33 + 1.33 - 0.33)kN = 4.33kN \]

\[ V_C = (1.67 - 1.33 + 0.33)kN = 0.67kN \]

\[ M_D = (3.33 + 5.33 - 1.33)kNm = 7.33kNm \]

One should remember that the method of superposition is valid only for linear elastic cases with small displacements only. So, prior to using influence lines in this way it is necessary to check that these conditions are satisfied.

It may seem that we can solve for these forces under the specified load case using equilibrium equations directly, and influence lines are not necessary. However, there may be requirement for obtaining these responses for multiple and more complex loading cases. For example, if we need to analyse for ten loading cases, it will be quicker to find only three influence lines and not solve for ten equilibrium cases.

The most important use of influence line is finding out the location of a load for which certain response will have a maximum value. For example, we may need to find the location of a moving load (say a gantry) on a beam (say a gantry girder) for which we get the maximum bending moment at a certain point. We can consider bending moment at point D of
Example 3, where the beam $AB$ becomes our gantry girder. Looking at the influence line of one can say that will reach its maximum value when the load is at point $D$. Influence lines can be used not only for concentrated forces, but for distributed forces as well, which is discussed in the next section.

4 Using Influence Lines for Uniformly Distributed Load

Consider the simply-supported beam $AB$ in Figure 6.5, of which the portion $CD$ is acted upon by a uniformly distributed load of intensity $w/\text{unit length}$. We want to find the value of a certain response function $R$ under this loading and let us assume that we have already constructed the influence line of this response function. Let the ordinate of the influence line at a distance $x$ from support $A$ be $F_R(x)$. If we consider an elemental length $dx$ of the beam at a distance $x$ from $A$, the total force acting on this elemental length is $wdx$. Since $dx$ is infinitesimal, we can consider this force to be a concentrated force acting at a distance $x$.

The contribution of this concentrated force $wdx$ to $R$ is:

$$dR = (wdx)F_R(x)$$

Therefore, the total effect of the distributed force from point $C$ to $D$ is:

$$R = \int_C^D dR = \int_{x_1}^{x_2} wF_R(x)dx$$

$$= w \int_{x_1}^{x_2} F_R(x)dx = \omega \text{ (area under the influence line from C to D)}$$

---

**Figure 5 Using influence line for a uniformly distributed loading**
Thus, we can obtain the response parameter by multiplying the intensity of the uniformly distributed load with the area under the influence line for the distance for which the load is acting. To illustrate, let us consider the uniformly distributed load on a simply supported beam (Figure 6). To find the vertical reaction at the left support, we can use the influence line for that we have obtained in Example 1. So we can calculate the reaction as:

\[ R_A = 2\text{kN/m} \times 0.5(3/4 + 1/4) \times 4\text{m} = 4\text{ kN} \]

![Figure 6.6 Uniformly distributed load acting on a beam](image)

Similarly, we can find any other response function for a uniformly distributed loading using their influence lines as well. For non-uniformly distributed loading, the intensity \( w \) is not constant through the length of the distributed load. We can still use the integration formulation:

\[ R = \int_C^D dR = \int_{x_1}^{x_2} wF_R(x)\,dx \]

However, we cannot take the intensity \( w \) outside the integral, as it is a function of \( x \).
UNIT – 2

SLOPE DEFLECTION METHOD

In this method the end moments or support moments expressed in terms of slopes, deflections, stiffness and length of the members. The unknown slope values (slopes) are determined from the condition of equilibrium of joints for moments that is

\[ M_{BA} + M_{BC} = 0 \]

![Slope Deflection Method Diagram](image)

Assumption

1. All the joints of the frame are rigid that is angle between the members do not change at a joint even after deformation.
2. The joints are assumed to rotate as a whole
3. Directions due to axial and shear stress are neglected because they are negligible or small

Sign Conventions

1. **Moments:**
   - Clockwise moment +ve
   - Anticlockwise moment –ve

2. **Rotation:**
   - Clockwise Rotation +ve
   - Anticlockwise Rotation –ve

3. **Sinking of support**
   - If right support sinks down is +ve
   - If left support sinks down is –ve

4. **Bending Moments**
   - Sagging BM is +ve and Hogging BM is –ve

5. **Shear Force**
   - Left side upward the SF is +ve
   - Left side downward the SF is -ve
   - Right side upward the SF is -ve
   - Right side downwardward the SF is +ve
1. Draw BMD, Elastic curve and SFD by slope deflection Method

Fixed End Moment:

\[ M_{FAB} = -\frac{w_{AB} b^2}{l^2} = -\frac{40 \times 3 \times 2^2}{5^2} = -19.2 \text{ kN-m} \]

\[ M_{FBA} = \frac{w_{BA} a^2}{l^2} = \frac{40 \times 2 \times 3^2}{5^2} = 28.8 \text{ kN-m} \]

\[ M_{FBC} = -\frac{w_{BC} l^2}{12} = -\frac{10 \times 6^2}{12} = -30 \text{ kN-m} \]

\[ M_{FCB} = \frac{w_{CB} l^2}{12} = \frac{10 \times 6^2}{12} = 30 \text{ kN-m} \]

Slope Deflection Equation

\[ M_{AB} = M_{FAB} + \frac{2EI}{l}(2\theta_A + \theta_B) \]

\[ = -19.2 + \frac{2 \times 1.5EI}{5} (0 + \theta_B) \quad (\theta_A = 0 \text{ due to fixity at support A}) \]

\[ M_{AB} = -19.2 + 0.6EI \theta_B \]

\[ M_{BA} = M_{FBA} + \frac{2EI}{l}(2\theta_A + \theta_B) \]

\[ = 28.8 + \frac{2 \times 1.5EI}{5} (0 + \theta_A) \]

\[ M_{BA} = 28.8 + 1.2EI \theta_A \]

\[ M_{BC} = M_{FBC} + \frac{2EI}{l}(2\theta_B + \theta_C) \]

\[ = -30 + \frac{2 \times 2EI}{6} (2\theta_B + \theta_C) \]

\[ M_{BC} = -30 + 1.33EI \theta_B + 0.667EI \theta_C \]

\[ M_{CB} = M_{FCB} + \frac{2EI}{l}(2\theta_C + \theta_B) \]

\[ = 30 + \frac{2 \times 2EI}{6} (2\theta_C + \theta_B) \]

\[ M_{CB} = 30 + 1.33EI \theta_C + 0.667EI \theta_B \]
Apply the condition of Equilibrium

\[ M_{BA} + M_{BC} = 0 \quad \text{and} \quad M_{CB} = 0 \]

At 'B'

\[ M_{BA} + M_{BC} = 0 \]

\[ 28.8 + 1.2EI \theta_B - 30 + 1.33EI \theta_B + 0.667EI \theta_C = 0 \]

\[ 2.533EI \theta_B + 0.667EI \theta_C = 1.2 \quad \rightarrow 1 \]

At 'C'

\[ M_{CB} = 0 \]

\[ 30 + 0.667EI \theta_C + 0.667EI \theta_B = 0 \]

\[ 0.667EI \theta_B + 1.33EI \theta_C = -30 \quad \rightarrow 2 \]

Solving Eq 1 and 2

\[ \theta_B = \frac{7.372}{EI} \quad \theta_C = \frac{-26.194}{EI} \]

Substitute the above values in the S-D Equation

\[ M_{AB} = -19.2 + 0.6EI \theta_B \]

\[ M_{AB} = -14.76 \text{ kN-m} \]

\[ M_{BA} = 28.8 + 1.2 EI \theta_B \]

\[ M_{BA} = 37.68 \text{ kN-m} \]

\[ M_{BC} = -30 + 1.33EI \theta_B + 0.667EI \theta_C \]

\[ M_{BC} = -37.68 \text{ kN-m} \]

\[ M_{CB} = 30 + 1.33EI \theta_C + 0.667EI \theta_B \]

\[ M_{CB} = 0 \]

Support Reaction
\[ V_A = 11.424\,\text{kN}, \quad V_{B1} = 28.576\,\text{kN}, \quad V_{B2} = 36.28\,\text{kN} \quad \text{and} \quad V_C = 23.72\,\text{kN} \]

\[ V_B = V_{B1} + V_{B2} = 58.734\,\text{kN} \]

**SFD**

**BMD**

**Elastic Curve**

2. Draw BMD, Elastic curve and SFD by slope deflection Method

Fixed End Moment:

\[ M_{FAB} = -\frac{wa^2}{l^2} = -\frac{49 \times 1 \times 2^2}{3^2} = -17.78 \, \text{kN-m} \]

\[ M_{FBA} = \frac{wa^2}{l^2} = \frac{40 \times 2 \times 1^2}{3^2} = 8.89 \, \text{kN-m} \]
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\[ M_{FBC} = -\frac{wL^2}{12} = -\frac{26 \times 4^2}{12} = -26.67 \text{ kN-m} \]

\[ M_{FCB} = \frac{wL^2}{12} = \frac{26 \times 4^2}{12} = 26.67 \text{ kN-m} \]

\[ M_{FCD} = M_{CD} = -10 \times 2 = -20 \text{ kN-m} \]

**Slope Deflection Equation**

\[ M_{AB} = M_{FAB} + \frac{2EI}{l} (2\theta_A + \theta_B) \]

\[ = -17.78 + \frac{2 \times 2 \times 2}{3} \left( 2\theta_A + \theta_B \right) \]

\[ (\theta_A = 0 \text{ due to fixity at support A}) \]

\[ M_{AB} = -17.78 + 1.33EI \theta_B \]

\[ M_{BA} = M_{FBA} + \frac{2EI}{l} (2\theta_A + \theta_B) \]

\[ = 8.89 + \frac{2 \times 2 \times 2}{3} \left( 2\theta_B + \theta_A \right) \]

\[ M_{BA} = 8.89 + 2.67EI \theta_B \]

\[ M_{BC} = M_{FBC} + \frac{2EI}{l} (2\theta_B + \theta_C) \]

\[ = -26.67 + \frac{2 \times 1.5 \times 2}{4} \left( 2\theta_B + \theta_C \right) \]

\[ M_{BC} = -26.67 + 1.5EI \theta_B + 0.75EI \theta_C \]

\[ M_{CB} = M_{FCB} + \frac{2EI}{l} (2\theta_C + \theta_B) \]

\[ = 26.67 + \frac{2 \times 1.5 \times 2}{4} \left( 2\theta_C + \theta_B \right) \]

\[ M_{CB} = 26.67 + 1.5EI \theta_C + 0.75EI \theta_B \]

Apply the condition of Equilibrium

\[ M_{BA} + M_{BC} = 0 \text{ and } M_{CB} + M_{CD} = 0 \]

At ‘B’

\[ M_{BA} + M_{BC} = 0 \]

\[ 8.89 + 2.67EI \theta_B - 26.67 + 1.5EI \theta_B + 0.75EI \theta_C = 0 \]

\[ 4.17EI \theta_B + 0.75EI \theta_C = 17.78 \]
At 'c'

\[ M_{CB} + M_{CD} = 0 \]

\[ 26.67 + 1.5EI \theta_C + 0.75EI \theta_B - 20 = 0 \]

\[ 0.75EI \theta_B + 1.5EI \theta_C = -6.67 \]

Solving Eq 1 and 2

\[ \theta_B = \frac{5592}{EI} \]

\[ \theta_C = \frac{-7.241}{EI} \]

Substitute the above values in the S-D Equation

\[ M_{AB} = -17.78 + 1.33EI \theta_B \]

\[ M_{AB} = -10.3426 \text{kN-m} \]

\[ M_{BA} = 8.89 + 2.67EI \theta_B \]

\[ M_{BA} = 23.703 \text{kN-m} \]

\[ M_{BC} = -26.67 + 1.5EI \theta_B + 0.75EI \theta_C \]

\[ M_{BC} = -23.703 \text{kN-m} \]

\[ M_{CB} = 26.67 + 1.5EI \theta_C + 0.75EI \theta_B \]

\[ M_{CB} = 20 \text{kN-m} \]

\[ M_{CD} = -20 \text{kN-m} \]

Support Reaction

\[ V_A = 22.21 \text{kN}, \quad V_{B1} = 17.79 \text{kN}, \quad V_{B2} = 40.935 \text{kN} \quad \text{and} \quad V_C = 49.065 \text{kN} \]

\[ V_B = V_{B1} + V_{B2} = 58.734 \text{kN} \]
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SINKING OF SUPPORT

\[ M_{AB} = \frac{-6EI\delta}{l^2} \]

\[ M_{BA} = \frac{-6EI\delta}{l^2} \]

**Slope Deflection equations**

\[ M_{AB} = M_{FAB} + \frac{2EI}{l} (2\theta_A + \theta_B - \frac{3\delta}{l}) \]

\( \delta \) is +ve when right side support sinks
\( \delta \) is -ve when left side support sinks

1. Analyse the continuous beam by slope deflection method the support B sinks by 5mm.
   Draw BMD, EC and SFD. Take \( EI = 2 \times 10^4 \) kN-mm

   **Fixed End Moment:**

   \[ M_{FAB} = \frac{-wL^2B^2}{l^2} = -\frac{60 \times 3 \times 2^2}{5^2} = -28.8 \text{ kN-m} \]
   \[ M_{FBA} = \frac{wL^2A^2}{l^2} = \frac{60 \times 2 \times 3^2}{5^2} = 43.2 \text{ kN-m} \]
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\[ M_{FB} = \frac{-pa b^2}{l^2} = \frac{-80 \times 2 \times 2^2}{4^2} = -40 \text{ kN-m} \]

\[ M_{FC} = \frac{-wb a^2}{l^2} = \frac{80 \times 2 \times 2^2}{4^2} = 40 \text{ kN-m} \]

\[ M_{FC} = M_{CD} = -20 \times 2 \times 1 = -40 \text{ kN-m} \]

Slope Deflection Equation

\[ M_{AB} = M_{FAB} + \frac{2EI}{l} (2\theta_A + \theta_B - \frac{3(+)\theta}{l}) \]

\[ = -28.8 + \frac{2 \times 2 \times 10^4}{5} (2\theta_A + \theta_B - \frac{3(0.005)}{5}) \quad (\theta_A = 0 \text{ due to fixity at support A}) \]

\[ M_{AB} = -52.8 + 0.8 \times 10^4 \theta_B \]

\[ M_{BA} = M_{FBA} + \frac{2EI}{l} (2\theta_A + \theta_B - \frac{3\delta}{l}) \]

\[ = 43.2 + \frac{2 \times 2 \times 10^4}{5} (2\theta_B + \theta_A - \frac{3(0.005)}{5}) \]

\[ M_{BA} = 19.2 + 1.6 \times 10^4 \theta_B \]

\[ M_{BC} = M_{FBC} + \frac{2EI}{l} (2\theta_B + \theta_C - \frac{3\delta}{l}) \]

\[ = -40 + \frac{2 \times 2 \times 10^4}{4} (2\theta_B + \theta_C - \frac{3(-0.005)}{5}) \]

\[ M_{BC} = -2.5 + 2 \times 10^4 \theta_B + 10^4 \theta_C \]

\[ M_{CB} = M_{FCB} + \frac{2EI}{l} (2\theta_C + \theta_B - \frac{3\delta}{l}) \]

\[ = 40 + \frac{2 \times 1.5EI}{4} (2\theta_C + \theta_B - \frac{3(-0.005)}{5}) \]

\[ M_{CB} = 77.5 + 2 \times 10^4 \theta_C + 10^4 \theta_B \]

Apply the condition of Equilibrium

\[ M_{BA} + M_{BC} = 0 \quad \text{and} \quad M_{CB} + M_{CD} = 0 \]

At ‘B’

\[ M_{BA} + M_{BC} = 0 \]

\[ 19.2 + 1.6 \times 10^4 \theta_B - 2.5 + 2 \times 10^4 \theta_B + 10^4 \theta_C = 0 \]

\[ 3.6 \times 10^4 \theta_B + 10^4 \theta_C = -16.7 \]
At 'c'

\[ M_{CB} + M_{CD} = 0 \]

\[ 77.5 + 2 \times 10^4 \theta_C + 10^4 \theta_B - 40 = 0 \]

\[ 10^4 \theta_B + 2 \times 10^4 \theta_C = -37.5 \]

Solving Eq 1 and 2

\[ \theta_B = 0.66 \times 10^{-4} \quad \theta_C = -19.1 \times 10^{-4} \]

Substitute the above values in the S-D Equation

\[ M_{AB} = -52.8 + 0.8 \times 10^4 \theta_B \]
\[ M_{AB} = -52.275 \text{ kN-m} \]

\[ M_{BA} = 19.2 + 1.6 \times 10^4 \theta_B \]
\[ M_{BA} = 20.256 \text{ kN-m} \]

\[ M_{BC} = -2.5 + 2 \times 10^4 \theta_B + 10^4 \theta_C \]
\[ M_{BC} = -20.256 \text{ kN-m} \]

\[ M_{CB} = 77.5 + 2 \times 10^4 \theta_C + 10^4 \theta_B \]
\[ M_{CB} = 40 \text{ kN-m} \]

\[ M_{CD} = -40 \text{ kN-m} \]

Support Reaction

\[ V_A = 30.4 \text{ kN}, \quad V_{B1} = 29.6 \text{ kN}, \quad V_{B2} = 35.064 \text{ kN} \quad \text{and} \quad V_C = 84.94 \text{ kN} \]

\[ V_B = V_{B1} + V_{B2} = 64.664 \text{ kN} \]
The image contains diagrams of Structural Analysis, including SFD (Shear Force Diagram) and BMD (Bending Moment Diagram). The SFD diagram shows shear forces at points A, B, C, and D, with values 30.4, 35.064, 35.064, and 40 respectively. The BMD diagram displays bending moments with values such as 52.275 kN-m, 72 kN-m, 80 kN-m, and 40 kN-m. The Elastic Curve is also illustrated, depicting the deflection of the structure.
**Analysis of NON-SWAY Portal Frames**

A frame is a structure having both horizontal and vertical members, such as beams and columns. The joint between any two members is assumed to rotate has a whole when loads are applied (rigid) hence they are called rigid jointed frames.

The frames is which the beams and columns are perpendicular to each other are called orthogonal frames. The moment of the joints in frames in the lateral direction is called Lateral Sway or Sway.

The frames which do not sway in lateral direction are called non sway portal frames.

Examples:
1. Analyse the frame shown in the figure and draw BMD

\[ \text{Fixed End Moment:} \]

\[ M_{FAB} = 0 \]
\[ M_{FBA} = 0 \]
\[ M_{FBC} = \frac{-40 \times 6^2}{12} = -120 \text{ kN-m} \]
\[ M_{FCB} = \frac{40 \times 6^2}{12} = 120 \text{ kN-m} \]
\[ M_{FCD} = M_{FDC} = 0 \]

\[ \text{Slope Deflection Equation} \]

\[ M_{AB} = M_{FAB} + \frac{2EI}{l} (2\theta_A + \theta_B) \]
\[ = 0 + \frac{2EI}{4} (2\theta_A + \theta_B) \quad (\theta_A = 0 \text{ due to fixity at support A}) \]
\[ M_{AB} = 0.5EI \theta_B \]

\[ M_{BA} = M_{FBA} + \frac{2EI}{l} (2\theta_A + \theta_B) \]
\[ = 0 + \frac{2EI}{4} (2\theta_B + \theta_A) \]
\[ M_{BA} = EI \theta_B \]

\[ M_{BC} = M_{FBC} + \frac{2EI}{l} (2\theta_B + \theta_C) \]
\[ = -120 + \frac{2 \times 2EI}{6} (2\theta_B + \theta_C) \]
\[ M_{BC} = -120 + 1.33EI \theta_B + 0.667EI \theta_C \]
\[ M_{CB} = M_{FCB} + \frac{2EI}{l} (2\theta_C + \theta_B) \]

\[ = 120 + \frac{2 \times 2EI}{6} (2\theta_C + \theta_B) \]

\[ M_{CB} = 120 + 1.33EI \theta_C + 0.667EI \theta_B \]

\[ M_{CD} = M_{FCD} + \frac{2EI}{l} (2\theta_C + \theta_D) \]

\[ = 0 + \frac{2EI}{4} (2\theta_C + \theta_D) \]

\[ M_{CD} = EI \theta_C \]

\[ M_{DC} = M_{FDC} + \frac{2EI}{l} (2\theta_D + \theta_C) \]

\[ = 0 + \frac{2EI}{4} (2\theta_D + \theta_C) \]

\[ M_{DC} = 0.5EI \theta_C \]

Apply the condition of Equilibrium

\[ M_{BA} + M_{BC} = 0 \] and \[ M_{CB} + M_{CD} = 0 \]

At ‘B’

\[ M_{BA} + M_{BC} = 0 \]

\[ EI \theta_B - 120 + 1.33EI \theta_B + 0.667EI \theta_C = 0 \]

\[ 2.33EI \theta_B + 0.667EI \theta_C = 120 \] \[ \rightarrow 1 \]

At ‘C’

\[ M_{CB} + M_{CD} = 0 \]

\[ 120 + 1.33EI \theta_C + 0.667EI \theta_B + EI \theta_C = 0 \]

\[ 0.667EI \theta_B + 2.33EI \theta_C = -120 \] \[ \rightarrow 2 \]

Solving Eq 1 and 2

\[ \theta_B = \frac{5.592}{EI} \]

\[ \theta_C = \frac{-7.241}{EI} \]
Substitute the above values in the S-D Equation

\[ M_{AB} = 0.5EI \theta_B \]
\[ M_{AB} = 36.14 \text{ kN-m} \]

\[ M_{BA} = EI \theta_B \]
\[ M_{BA} = 72.28 \text{ kN-m} \]

\[ M_{BC} = -120 + 1.33EI \theta_B + 0.667EI \theta_C \]
\[ M_{BC} = -71.6 \text{ kN-m} \]

\[ M_{CB} = 120 + 1.33EI \theta_C + 0.667EI \theta_B \]
\[ M_{CB} = 72.3 \text{ kN-m} \]

\[ M_{CD} = EI \theta_C \]
\[ M_{CD} = -72.26 \text{ kN-m} \]

\[ M_{DC} = 0.5EI \theta_C \]
\[ M_{DC} = -36.14 \text{ kN-m} \]
2. Draw BMD and EC by slope deflection for the frame shown in figure.

![Frame Diagram]

**Fixed End Moment:**

\[ M_{FAB} = 0 \]
\[ M_{FBA} = 10 \times 3 \times 1.5 = 45\text{N} \times \text{m} \]
\[ M_{FBC} = \frac{-wab^2}{l^2} - \frac{-wab^2}{l^2} = -\frac{30 \times 1 \times 3^2}{4^2} - \frac{30 \times 3 \times 1^2}{4^2} = -22.5\text{ kN} \times \text{m} \]
\[ M_{FCB} = \frac{wba^2}{l^2} + \frac{wba^2}{l^2} = \frac{30 \times 1 \times 3^2}{4^2} + \frac{30 \times 3 \times 1^2}{4^2} = 22.5\text{ kN} \times \text{m} \]
\[ M_{FBD} = \frac{-wab^2}{l^2} = -\frac{20 \times 2 \times 2^2}{5^2} = 9.6\text{ kN} \times \text{m} \]
\[ M_{FDB} = \frac{-wba^2}{l^2} = -\frac{20 \times 2 \times 3^2}{5^2} = -14.4\text{ kN} \times \text{m} \]

**Slope Deflection Equation**

\[ M_{BC} = M_{FBC} + \frac{2EI}{l}(2\theta_B + \theta_C) \]
\[ = -22.5 + \frac{2EI}{4} (2\theta_B + \theta_C) \]
\[ M_{BC} = -22.5 + 0.5EI \theta_B \]

\[ M_{CB} = M_{FCB} + \frac{2EI}{l}(2\theta_C + \theta_B) \]
\[ = 22.5 + \frac{2EI}{4} (2\theta_C + \theta_B) \]
\[ M_{CB} = 22.5 + 0.5EI \theta_B \]
\[ M_{BD} = M_{FCD} + \frac{2EI}{t} (2\theta_B + \theta_D) \]
\[ = 9.6 + \frac{2EI}{5} (2\theta_B + \theta_D) \]
\[ M_{BD} = 9.6 + 0.8EI \theta_B \]

\[ M_{DB} = M_{FDC} + \frac{2EI}{t} (2\theta_D + \theta_B) \]
\[ = 0 + \frac{2EI}{5} (2\theta_D + \theta_B) \]
\[ M_{DB} = -14.4 + 0.4EI \theta_D \]

Apply the condition of Equilibrium

At ‘B’
\[ M_{BA} + M_{BC} + M_{BD} = 0 \]
\[ 45 - 22.5 + EI \theta_B + 9.6 + 0.8EI \theta_B = 0 \]
\[ 1.8EI \theta_B = -32.1 \]
\[ \theta_B = \frac{-32.1}{1.8EI} \]
\[ \theta_B = \frac{-17.83}{EI} \]

Substitute the above value in the S-D Equation
\[ M_{BA} = 45\text{kN-m} \]
\[ M_{BC} = -22.5 + EI \theta_B \]
\[ M_{BC} = -40.33\text{ kN-m} \]
\[ M_{CB} = 22.5 + 0.5EI \theta_B \]
\[ M_{CB} = 13.585\text{ kN-m} \]
\[ M_{BD} = 9.6 + 0.8EI \theta_B \]
\[ M_{BD} = -4.67\text{ kN-m} \]
\[ M_{DB} = -14.4 + 0.4EI \theta_B \]
\[ M_{DB} = -21.52\text{ kN-m} \]
Consider a continuous beam ABC as shown in the figure. The final moment developed at the intermediate support B should follow the condition $M_{BA} + M_{BC} = 0$ but the fixed end moments (FEM) at that support will be $M_{FBA} + M_{FBC} = 0$.

The algebraic sum of the fixed end moments (FEM) thus obtained is the unbalanced moment at B. This unbalanced moment is balanced and redistributed to both $M_{FBA}$ and $M_{FBC}$ depending upon the stiffness of the member. Half of the distributed moment will be carried over to the far end. Hence we will again have unbalanced moments, so that we have to balance and distribute.

This iteration process will continue till we get balanced moments that is $M_{BA} + M_{BC} = 0$, similarly at the last simple support $M_{CB} = 0$.

This iterative method of balancing and redistributing of unbalanced moments to obtain the final balanced moments is called moment distribution method.

In order to distribute the moments we should calculate stiffness factors and distribution factors.

**Stiffness Factor (K):**

Stiffness factor depends upon the support condition at the far end.

- If the far end is continuous or fixed $K = \frac{4EI}{l}$
- If the far end is simply support or discontinuous $K = \frac{3EI}{l}$

Ex: $K_{AB} = \frac{4EI}{l}$, $K_{BA} = \frac{4EI}{l}$, $K_{BC} = \frac{4EI}{l}$, $K_{CB} = \frac{4EI}{l}$

**Distribution Factor (Δ):**

The unbalanced moments are distributed only at the intermediate supports, hence distribution factors are calculated only at the intermediate supports.

$$BA = \frac{K_{BA}}{K_{BA} + K_{BC}} \quad \text{or} \quad \frac{K_{BA}}{2K_{B}}$$

$$BC = \frac{K_{BC}}{K_{BA} + K_{BC}} \quad \text{or} \quad \frac{K_{BC}}{K_{B}}$$
1. Analyse continuous beam shown in the figure by moment distribution method. Draw BMD, EC and SFD.

\[ F = 50\text{kN} \]
\[ w = 15\text{kN/m} \]
\[ F = 80\text{kN} \]

\[ A \quad B \quad C \quad D \]
\[ 2\text{m} \quad 2\text{m} \quad 5\text{m} \quad 2\text{m} \quad 3\text{m} \]

**Fixed End Moments:**

- \[ M_{FAB} = \frac{-w_0 a^2}{l^2} = \frac{-50 \times 2 \times 2^2}{4^2} = -25 \text{kN-m} \]
- \[ M_{FBA} = \frac{w_0 a^2}{l^2} = \frac{50 \times 2 \times 2^2}{4^2} = 25 \text{kN-m} \]
- \[ M_{FBC} = \frac{-w_0 l^2}{12} = \frac{-15 \times 5^2}{12} = -31.25 \text{kN-m} \]
- \[ M_{FCB} = \frac{w_0 l^2}{12} = \frac{15 \times 5^2}{12} = 31.25 \text{kN-m} \]
- \[ M_{FCD} = \frac{-w_0 h^2}{l^2} = \frac{-80 \times 2 \times 3^2}{5^2} = -57.6 \text{kN-m} \]
- \[ M_{FDC} = \frac{w_0 h^2}{l^2} = \frac{80 \times 3 \times 2^2}{5^2} = 38.4 \text{kN-m} \]

**Stiffness Factor (K)**

- \[ K_{BA} = \frac{4EI}{l} = \frac{4EI}{4} = EI \]
- \[ K_{BC} = \frac{4EI}{l} = \frac{4EI}{5} = 0.8EI \]
- \[ K_{CB} = \frac{4EI}{l} = \frac{4EI}{5} = 0.8EI \]
- \[ K_{CD} = \frac{4EI}{l} = \frac{4EI}{5} = 0.8EI \]

**Distribution Factor (Δ)**

- \[ \Delta_{BA} = \frac{K_{BA}}{K_{BA} + K_{BC}} = \frac{EI}{EI + 0.8EI} = 0.56 \]
- \[ \Delta_{CB} = \frac{K_{CB}}{K_{CB} + K_{CD}} = \frac{0.8EI}{0.8EI + 0.8EI} = 0.5 \]
- \[ \Delta_{BC} = \frac{K_{BC}}{K_{BA} + K_{BC}} = \frac{0.8EI}{0.8EI + EI} = 0.44 \]
- \[ \Delta_{CD} = \frac{K_{CD}}{K_{CD} + K_{CB}} = \frac{0.8EI}{0.8EI + 0.8EI} = 0.5 \]
### Moment Distribution Table

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>FEM</strong></td>
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<td>-31.25</td>
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<tr>
<td><strong>Carry over</strong></td>
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<td>-0.0375</td>
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<tr>
<td><strong>Carry over</strong></td>
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<td>-0.018</td>
<td>-0.079</td>
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<tr>
<td><strong>Balance</strong></td>
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<td>+0.008</td>
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<tr>
<td><strong>Carry over</strong></td>
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<td>0.004</td>
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<td></td>
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<td></td>
</tr>
<tr>
<td><strong>Final Moment</strong></td>
<td>-25</td>
<td>24.8</td>
<td>-24.8</td>
<td>44.39</td>
<td>-44.38</td>
</tr>
</tbody>
</table>

### Support Reaction

**VA** = 25.05kN,  **VB1** = 24.95kN,  **VB2** = 58.55kN,  **VC1** = 41.42kN,  **VC2** = 47.878kN

\[ V_B = V_{BI} + V_{B2} = 25.05 + 24.95 = 50.00 \text{kN} \]
\[ V_C = V_{C1} + V_{C2} = 41.42 + 47.878 = 89.298 \text{kN} \]
2. Analyse continuous beam shown in the figure by moment distribution method. Draw BMD, EC and SFD.

Fixed End Moments:

\[ M_{FAB} = \frac{-wA h^2}{l^2} = \frac{-40 \times 3 \times 2^2}{5^2} = -19.2 \text{ kN-m} \]

\[ M_{FBA} = \frac{wb a^2}{l^2} - \frac{40 \times 2 \times 3^2}{5^2} = 88.8 \text{ kN-m} \]

\[ M_{FBC} = \frac{-wI^2}{12} = -10 \times 4^2 = -13.33 \text{ kN-m} \]

\[ M_{FCB} = \frac{wI^2}{12} = \frac{10 \times 4^2}{12} = 13.33 \text{ kN-m} \]

\[ M_{FCD} = \frac{-wI^2}{12} = -10 \times 5^2 = -20.83 \text{ kN-m} \]
\[ M_{FDC} = \frac{w l^2}{12} = \frac{10 \times 5^2}{12} = 20.83 \text{ kN-m} \]

**Stiffness Factor (K)**

\[ K_{BA} = \frac{4EI}{l} = \frac{4EI}{5} = 0.8EI \]

\[ K_{BC} = \frac{4EI}{l} = \frac{4EI}{4} = EI \]

\[ K_{CB} = \frac{4EI}{l} = \frac{4EI}{4} = EI \]

\[ K_{CD} = \frac{3EI}{l} = \frac{3EI}{5} = 0.6EI \]

**Distribution Factor (Δ)**

\[ \Delta_{BA} = \frac{K_{BA}}{K_{BA} + K_{BC}} = \frac{0.8EI}{0.8EI + EI} = 0.44 \]

\[ \Delta_{CB} = \frac{K_{CB}}{K_{CB} + K_{CD}} = \frac{EI}{EI + 0.6EI} = 0.625 \]

\[ \Delta_{BC} = \frac{K_{CB}}{K_{BC} + K_{CB}} = \frac{EI}{0.6EI + EI} = 0.375 \]

**Moment Distribution Table**

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Release D</td>
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</tr>
<tr>
<td>Carry over</td>
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<tr>
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<tr>
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</table>

**Support Reaction**

\[
\begin{align*}
V_A &= 17.15\text{kN}, \quad V_{B1} = 22.85\text{kN}, \quad V_{B2} = 19.1\text{kN}, \quad V_{C1} = 20.9\text{kN}, \quad V_{C2} = 29.5\text{kN} \\
V_B &= V_{B1} + V_{B2} = 41.95\text{kN} \\
V_C &= V_{C1} + V_{C2} = 50.40\text{kN}
\end{align*}
\]

**SFD**

**BMD**
3. Analyse continuous beam shown in the figure by moment distribution method. Draw BMD, EC and SFD.

Fixed End Moments:

\[ M_{FAB} = -\frac{wL^2}{12} = -\frac{20 \times 2^2}{12} = -106.67 \text{ kN-m} \]

\[ M_{FBA} = \frac{wL^2}{12} = \frac{20 \times 2^2}{12} = 106.67 \text{ kN-m} \]

\[ M_{FBC} = -\frac{wa^2}{l^2} = -\frac{60 \times 2 \times 2^2}{4^2} = -30 \text{ kN-m} \]

\[ M_{FCB} = \frac{wa^2}{l^2} = \frac{60 \times 2 \times 2^2}{4^2} = 30 \text{ kN-m} \]

\[ M_{FCD} = M_{CD} = -20 \times 2 = -40 \text{ kN-m} \]

Stiffness Factor (K)

\[ K_{BA} = \frac{4EI}{l} = \frac{4 \times 2EI}{8} = EI \]

\[ K_{BC} = \frac{3EI}{l} = \frac{3EI}{4} = 0.75EI \]

\[ K_{CB} = \frac{4EI}{l} = \frac{4EI}{4} = EI \]

\[ K_{CD} = 0 \]

Distribution Factor (λ)

\[ \lambda_{BA} = \frac{K_{BA}}{K_{BA} + K_{BC}} = \frac{EI}{EI + 0.75EI} = 0.57 \]

\[ \lambda_{CB} = \frac{K_{CB}}{K_{CB} + K_{CD}} = \frac{EI}{EI + 0} = 1 \]

\[ \lambda_{BC} = \frac{K_{BC}}{K_{BA} + K_{BC}} = \frac{0.75EI}{EI + 0.75EI} = 0.43 \]

\[ \lambda_{CD} = \frac{K_{CD}}{K_{CD} + K_{CB}} = 0 \]
### Moment Distribution Table

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>FEM</strong></td>
<td>-106.67</td>
<td>106.67</td>
<td>-30</td>
<td>30</td>
</tr>
<tr>
<td><strong>Balance OH</strong></td>
<td>0.57</td>
<td>0.43</td>
<td>1</td>
<td>0</td>
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<tr>
<td><strong>Carry over</strong></td>
<td>+10</td>
<td>+10</td>
<td>+10</td>
<td>+10</td>
</tr>
<tr>
<td><strong>Initial Moment</strong></td>
<td>-106.67</td>
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<td>-25</td>
<td>40</td>
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<tr>
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<td>-35.12</td>
<td>-35.12</td>
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<td><strong>Carry over</strong></td>
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<tr>
<td><strong>Final Moment</strong></td>
<td>-129.95</td>
<td>60.12</td>
<td>-60.12</td>
<td>40</td>
</tr>
</tbody>
</table>

### Support Reaction

\[ V_A = 88.73 \text{kN}, \quad V_{B1} = 71.27 \text{kN}, \quad V_{B2} = 35.03 \text{kN}, \quad V_C = 106.3 \text{kN} \]

\[ V_B = V_{B1} + V_{B2} = 41.95 \text{kN} \]

\[ V_{B1} = 71.27 \text{kN}, \quad V_{B2} = 35.03 \text{kN} \]

\[ V_A = 88.73 \text{kN} \]

### SFD

\[ SFD \]

\[ BMD \]
SINKING OF SUPPORT

1. Moments due to sinking

\[ M = -\frac{6EI\delta}{l^2} \]

If right support sinks down \( \delta \) is +ve
If left support sinks down \( \delta \) is -ve

2. Moments due to Rotation

Near end moment = \( \frac{4EI\theta}{l} \)
For far end moment = \( \frac{2EI\theta}{l} \)

Where \( \theta \) is rotation, clockwise +ve and anticlockwise -ve
The above should be added to the FEM

1. Analyse the continuous beam by moment distribution method. Support B yields by 9mm. Take \( EI = 1 \times 10^{12} \text{ N-mm}^2 \), draw BMD and EC

![Continuous Beam Diagram]

Fixed End Moments:

\[ M_{FAB} = \frac{-wb^2}{12} - \frac{6EI\delta}{l^2} - \frac{60 \times 2 \times 1^2}{3^2} - \frac{-6 \times 1 \times 10^3 \times 0.009}{3^2} = -19.33 \text{ kN-m} \]

\[ M_{FBA} = \frac{wb^2}{12} - \frac{6EI\delta}{l^2} - \frac{-60 \times 1 \times 2^2}{3^2} - \frac{-6 \times 1 \times 10^3 \times 0.009}{3^2} = 20.67 \text{ kN-m} \]

\[ M_{FBC} = \frac{-wL^2}{12} - \frac{-6EI(\delta)}{l^2} - \frac{-20 \times 6^2}{12} - \frac{-6 \times 1 \times 10^3 \times (-0.009)}{6^2} = -58.5 \text{ kN-m} \]

\[ M_{FCD} = M_{CD} = -20 \times 1 = -20 \text{ kN-m} \]

Stiffness Factor \( (K) \)

\[ K_{BA} = \frac{4EI}{l} = \frac{4EI}{3} = 1.33EI \]

\[ K_{BC} = \frac{3EI}{l} = \frac{3EI}{6} = 0.5EI \]

\[ K_{CB} = \frac{4EI}{l} = \frac{4EI}{6} = 0.667EI \]
\[ K_{CD} = 0 \]

**Distribution Factor (\( \Delta \))**

\[
\Delta_{BA} = \frac{K_{BA}}{K_{BA} + K_{BC}} = \frac{1.33EI}{1.33EI + 0.5EI} = 0.725
\]

\[
\Delta_{CB} = \frac{K_{BC}}{K_{BA} + K_{BC}} = \frac{0.5EI}{1.33EI + 0.5EI} = 0.275
\]

\[
\Delta_{BC} = \frac{K_{BC}}{K_{CB} + K_{CD}} = \frac{0.67EI}{0.67EI + 0} = 1
\]

\[
\Delta_{CD} = 0
\]

**Moment Distribution Table**

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
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<td>FEM</td>
<td>0.725</td>
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<tr>
<td>BalanceOH</td>
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<td>20.67</td>
<td>-58.5</td>
<td>61.5</td>
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<tr>
<td>Carry over</td>
<td></td>
<td></td>
<td>-41.5</td>
<td></td>
</tr>
<tr>
<td>Initial Moment</td>
<td>-19.33</td>
<td>20.67</td>
<td>-79.25</td>
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<tr>
<td>Balance</td>
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<td>-63.14</td>
<td>20</td>
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</tbody>
</table>

**Support Reaction**

\[ V_A = -1.8kN, \quad V_{B1} = 61.8kN, \quad V_{B2} = 67.19kN, \quad V_C = 72.81kN \]

\[ V_B = V_{B1} + V_{B2} = 128.99kN \]
Analysis of Non-Sway frames

1. Draw BMD, EC and SFD for frame shown in the figure.

Fixed End Moments:

\[ M_{FAB} = -\frac{w l^2}{12} = -\frac{20 \times 4^2}{12} = -26.67 \text{ kN-m} \]

\[ M_{FBA} = \frac{w l^2}{12} = \frac{20 \times 4^2}{12} = 26.67 \text{ kN-m} \]

\[ M_{FBC} = -\frac{w a b^2}{l^2} = -\frac{40 \times 2 \times 3^2}{5^2} = -28.8 \text{ kN-m} \]

\[ M_{FCB} = \frac{w b a^2}{l^2} = \frac{60 \times 3 \times 2^2}{5^2} = 19.2 \text{ kN-m} \]

\[ M_{FBD} = \frac{w a b^2}{l^2} = \frac{20 \times 2 \times 2^2}{4^2} = 10 \text{ kN-m} \]

\[ M_{FDB} = -\frac{w b a^2}{l^2} = -\frac{20 \times 2 \times 2^2}{4^2} = -10 \text{ kN-m} \]

Stiffness Factor (K)

\[ K_{BA} = \frac{4EI}{l} = \frac{4EI}{4} = EI \]

\[ K_{BD} = \frac{4EI}{l} = \frac{4EI}{4} = EI \]
\[ K_{BC} = \frac{3EI}{l} = \frac{3EI}{5} = 0.6EI \]

\[ \text{Distribution Factor(Δ)} = \frac{Δ_{BA}}{K_{BA} + K_{BC} + K_{BD}} = \frac{EI}{EI + 0.6EI + EI} = 0.385 \]

\[ BC = \frac{K_{BC}}{K_{BA} + K_{BC} + K_{BD}} = \frac{0.6EI}{EI + 0.6EI + EI} = 0.23 \]

**Moment Distribution Table**

<table>
<thead>
<tr>
<th></th>
<th>AB</th>
<th>BA</th>
<th>BC</th>
<th>CB</th>
<th>BD</th>
<th>DB</th>
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<tbody>
<tr>
<td>FEM</td>
<td>-26.67</td>
<td>26.67</td>
<td>-28.8</td>
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<td>BalanceOH</td>
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<td>-38.4</td>
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<td>-10</td>
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<tr>
<td>Carry over</td>
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<td>-9.6</td>
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<td>Initial Moment</td>
<td>-26.67</td>
<td>26.67</td>
<td>-38.4</td>
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<td>Balance</td>
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<tr>
<td>Final Moment</td>
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<td>27.33</td>
<td>-38</td>
<td>0</td>
<td>10.67</td>
<td>-9.67</td>
</tr>
</tbody>
</table>
UNIT – 4
SWAY ANALYSIS

Sway analysis by slope deflection method

1. Analyse the frame shown in the figure by slope deflection method Draw BMD and EC

\[
\begin{align*}
\text{Fixed End Moment:} \\
M_{FAB} &= 0 \\
M_{FBA} &= 0 \\
M_{FBC} &= -\frac{wL^2}{12} = -\frac{40 \times 6^2}{12} = -120 \text{ kN-m} \\
M_{FCB} &= \frac{wL^2}{12} = \frac{40 \times 6^2}{12} = 120 \text{ kN-m} \\
M_{FCD} &= M_{FDC} = 0
\end{align*}
\]

Slope Deflection Equation

\[
\begin{align*}
M_{AB} &= M_{FAB} + \frac{2EI}{t} \left(2\theta_A + \theta_B - \frac{3\delta}{L}\right) \\
&= 0 + \frac{2EI}{4} \left(2\theta_A + \theta_B - \frac{3\delta}{L}\right) \\
&= 0.5EI \theta_B - 0.375EI\delta \\
M_{BA} &= M_{FBA} + \frac{2EI}{t} \left(2\theta_A + \theta_B - \frac{3\delta}{L}\right) \\
&= 0 + \frac{2EI}{4} \left(2\theta_A + \theta_B - \frac{3\delta}{L}\right) \\
&= EI \theta_B - 0.375EI\delta
\end{align*}
\]
\[M_{BC} = M_{FBC} + \frac{2EI}{l}(2\theta_B + \theta_C - \frac{3\delta}{l})\]
\[= -120 + \frac{2 \times 2EI}{6} (2\theta_B + \theta_C - \frac{3\delta}{l})\]

\[M_{BC} = -120 + 1.33EI \theta_B + 0.667EI \theta_C\]

\[M_{CB} = M_{FCB} + \frac{2EI}{l}(2\theta_C + \theta_B - \frac{3\delta}{l})\]
\[= 120 + \frac{2 \times 2EI}{6} (2\theta_C + \theta_B - \frac{3\delta}{l})\]

\[M_{CB} = 120 + 1.33EI \theta_C + 0.667EI \theta_B\]

\[M_{CD} = M_{FCD} + \frac{2EI}{l}(2\theta_C + \theta_D - \frac{3\delta}{l})\]
\[= 0 + \frac{2 \times 2EI}{6} (2\theta_C + \theta_D - \frac{3\delta}{l})\]

\[M_{CD} = 1.33EI \theta_C - 0.33EI\delta\]

\[M_{DC} = M_{FDC} + \frac{2EI}{l}(2\theta_D + \theta_C - \frac{3\delta}{l})\]
\[= 0 + \frac{2 \times 2EI}{6} (2\theta_D + \theta_C - \frac{3\delta}{l})\]

\[M_{DC} = 0.67EI \theta_C - 0.33EI\delta\]

Apply the condition of Equilibrium

\[M_{BA} + M_{BC} = 0\] and \[M_{CB} + M_{CD} = 0\]

At ‘B’

\[M_{BA} + M_{BC} = 0\]

\[EI \theta_B - 0.375EI\delta - 120 + 1.33EI \theta_B + 0.667EI \theta_C = 0\]
\[2.33EI \theta_B + 0.667EI \theta_C - 0.375EI\delta = 120\] \[\rightarrow 1\]

At ‘C’

\[M_{CB} + M_{CD} = 0\]

\[120 + 1.33EI \theta_C + 0.667EI \theta_B + 0.67EI \theta_C - 0.33EI\delta = 0\]
\[0.667EI \theta_B + 2.33EI \theta_C - 0.33EI\delta = -120\] \[\rightarrow 2\]
Shear condition

\[ H_A = \frac{M_{AB} + M_{LA}}{4} \]

\[ H_A = 0.375EI\theta_B - 0.1875EI\delta \]

\[ H_D = 0.33EI\theta_C - 0.11EI\delta \]

\[ \Sigma H = 0 \]

\[ H_A + H_D = 0 \]

\[ 0.375EI\theta_B - 0.1875EI\delta + 0.33EI\theta_C - 0.11EI\delta = 0 \]

\[ 0.375EI\theta_B + 0.33EI\theta_C - 0.296EI\delta = 0 \]

Solving Eq 1, 2 and 3

\[ \theta_B = \frac{12.79}{EI} \]

\[ \theta_C = \frac{-60.14}{EI} \]

\[ \delta = \frac{24.83}{EI} \]

Substitute the above values in the S-D Equation

\[ M_{AB} = 0.5EI\theta_B - 0.375EI\delta \]

\[ M_{AB} = 27.31 \text{ kN-m} \]

\[ M_{BA} = EI\theta_B - 0.375EI\delta \]

\[ M_{BA} = 63.67 \text{ kN-m} \]

\[ M_{BC} = -120 + 1.33EI\theta_B + 0.667EI\theta_C \]

\[ M_{BC} = -63.65 \text{ kN-m} \]

\[ M_{CB} = 120 + 1.33EI\theta_C + 0.667EI\theta_B \]

\[ M_{CB} = 88.64 \text{ kN-m} \]

\[ M_{CD} = 1.33EI\theta_C - 0.33EI\delta \]

\[ M_{CD} = -88.1 \text{ kN-m} \]

\[ M_{DC} = 0.67EI\theta_C - 0.33EI\delta \]

\[ M_{DC} = -48.29 \text{ kN-m} \]
2. Analyse the frame shown in the figure by slope deflection method Draw BMD and EC

Fixed End Moment:

\[ M_{FAB} = \frac{-wa b^2}{l^2} = \frac{-40 \times 3 \times 1^2}{4} = -7.5 \text{ kN-m} \]

\[ M_{FBA} = \frac{wb a^2}{l^2} = \frac{40 \times 3 \times 3^2}{4^2} = 22.5 \text{ kN-m} \]

\[ M_{FBC} = \frac{-wa b^2}{l^2} = \frac{-80 \times 3 \times 3^2}{6^2} = -60 \text{ kN-m} \]

\[ M_{FCB} = \frac{wb a^2}{l^2} = \frac{80 \times 3 \times 3^2}{6^2} = 60 \text{ kN-m} \]
\[ M_{FCD} = M_{FDC} = 0 \]

**Slope Deflection Equation**

\[ M_{AB} = M_{FAB} + \frac{2EI}{l} (2\theta_A + \theta_B - \frac{3\delta}{l}) \]
\[ = -7.5 + \frac{2EI}{4} (2\theta_A + \theta_B - \frac{3\delta}{l}) \]
\[ (\theta_A = 0 \text{ due to fixity at support A}) \]

\[ [M_{AB}] = -7.5 + 0.5EI \theta_B - 0.375EI \delta \]

\[ M_{BA} = M_{FBA} + \frac{2EI}{l} (2\theta_A + \theta_B - \frac{3\delta}{l}) \]
\[ = 22.5 + \frac{2EI}{4} (2\theta_B + \theta_A - \frac{3\delta}{l}) \]

\[ [M_{BA}] = 22.5 + EI \theta_B - 0.375EI \delta \]

\[ M_{BC} = M_{FBC} + \frac{2EI}{l} (2\theta_B + \theta_C - \frac{3\delta}{l}) \]
\[ = -60 + \frac{2 \times 2EI}{6} (2\theta_B + \theta_C - \frac{3\delta}{l}) \]

\[ [M_{BC}] = -60 + 1.33EI \theta_B + 0.667EI \theta_C \]

\[ M_{CB} = M_{FCB} + \frac{2EI}{l} (2\theta_C + \theta_B - \frac{3\delta}{l}) \]
\[ = 60 + \frac{2 \times 2EI}{6} (2\theta_C + \theta_B - \frac{3\delta}{l}) \]

\[ [M_{CB}] = 60 + 1.33EI \theta_C + 0.667EI \delta_E \]

\[ M_{CD} = M_{FCD} + \frac{2 \times 2EI}{l} (2\theta_C + \theta_D - \frac{3\delta}{l}) \]
\[ = 0 + \frac{2EI}{4} (2\theta_C + \theta_D - \frac{3\delta}{l}) \]

\[ [M_{CD}] = EI \theta_C - 0.375EI \delta \]

\[ M_{DC} = M_{FDC} + \frac{2 \times 2EI}{l} (2\theta_D + \theta_C - \frac{3\delta}{l}) \]
\[ = 0 + \frac{2EI}{4} (2\theta_D + \theta_C - \frac{3\delta}{l}) \]

\[ [M_{DC}] = 0.5EI \theta_C - 0.375EI \delta \]
Apply the condition of Equilibrium

\( M_{BA} + M_{BC} = 0 \) and \( M_{CB} + M_{CD} = 0 \)

At 'B'

\[ M_{BA} + M_{BC} = 0 \]

\[ 22.5 + EI \theta_B - 0.375EI\delta - 60 + 1.33EI \theta_B + 0.667EI \theta_C = 0 \]

\[ 2.33EI \theta_B + 0.667EI \theta_C - 0.375EI\delta = 37.5 \] \( \rightarrow 1 \)

At 'C'

\[ M_{CB} + M_{CD} = 0 \]

\[ 60 + 1.33EI \theta_C + 0.667EI \theta_B - EI \theta_C - 0.375EI\delta = 0 \]

\[ 0.667EI \theta_B + 2.33EI \theta_C - 0.375EI\delta = -60 \] \( \rightarrow 2 \)

Shear condition

\[ H_A = \frac{M_{AB} + M_{AB} - (40 \times 1)}{4} \]

\[ H_A = 0.375EI\theta_B - 0.1875EI\delta - 6.25 \]

\[ H_D = \frac{M_{CD} + M_{CD}}{4} \]

\[ H_D = 0.33EI\theta_C - 0.185EI\delta \]

\[ \Sigma H = 0 \]

\[ H_A + H_D + 40 = 0 \]

\[ 0.375EI\theta_B - 0.1875EI\delta - 6.25 + 0.33EI\theta_C - 0.185EI\delta + 40 = 0 \]

\[ 0.375EI \theta_B + 0.375EI \theta_C - 0.375EI\delta = -33.75 \] \( \rightarrow 3 \)
Solving Eq 1, 2 and 3

\[ \theta_B = \frac{39.36}{EI} \quad \theta_C = \frac{-19.36}{EI} \quad \delta = \frac{110}{EI} \]

Substitute the above values in the S-D Equation

\[ M_{AB} = -7.5 + 0.5EI \theta_B - 0.375EI\delta \]
\[ M_{AB} = -29.07 \text{kN-m} \]

\[ M_{BA} = 22.5 + EI \theta_B - 0.375EI\delta \]
\[ M_{BA} = 20.61 \text{kN-m} \]

\[ M_{BC} = -60 + 1.33EI \theta_B + 0.667EI \theta_C \]
\[ M_{BC} = -20.62 \text{kN-m} \]

\[ M_{CB} = 60 + 1.33EI \theta_C + 0.667EI \theta_B \]
\[ M_{CB} = 60.62 \text{kN-m} \]

\[ M_{CD} = EI \theta_C - 0.33EI\delta \]
\[ M_{CD} = -60.61 \text{kN-m} \]

\[ M_{DC} = 0.5EI \theta_C - 0.33EI\delta \]
\[ M_{DC} = -50.93 \text{kN-m} \]
Sway analysis by Moment Distribution method

In moment distribution method of sway frames, we have to carry out two analysis to get final moments. They are

1. Non sway analysis
2. Sway analysis

Final Moment = Non sway moment + (C x Sway moment)

1. Analyse the frame by moment distribution method.

Non Sway Analysis

The sway in the frame is prevented by the force δ. As there are no loads on the members, there are no FEM’s developed. Hence the final moments for non sway analysis will be absent.

Apply the shear conditions to the vertical members

$$\Sigma_H = 0,$$

$$H_A + H_D + 80 - \delta = 0, \quad \delta = 80\text{kN} \quad [H_A = H_D = 0]$$

Sway Analysis

Fixed End Moments:

$$M_{FAB} = M_{FBA} = \frac{6EI\delta}{l^2} = \frac{-6\times 2EI\delta}{l^2}$$

$$M_{FCD} = M_{CD} = \frac{-6EI\delta}{l^2} = \frac{-6\times 1.5EI\delta}{l^2}$$
Stiffness Factor ($K$)

\[
\begin{align*}
K_{BA} &= \frac{4EI}{l} - \frac{4 \times 2EI}{6} = 1.33EI \\
K_{BC} &= \frac{4EI}{l} - \frac{4EI}{4} = EI \\
K_{CB} &= \frac{4EI}{l} - \frac{4EI}{4} = EI \\
K_{CD} &= \frac{4EI}{l} - \frac{4 \times 1.5EI}{4} = 1.5EI
\end{align*}
\]

Distribution Factor ($\Delta$)

\[
\begin{align*}
\Delta_{BA} &= \frac{K_{BA}}{K_{BA} + K_{BC}} = 0.57 \\
\Delta_{CB} &= \frac{K_{CB}}{K_{CB} + K_{CD}} = 0.4 \\
\Delta_{BC} &= \frac{K_{BC}}{K_{BA} + K_{BC}} = 0.43 \\
\Delta_{CD} &= \frac{K_{CD}}{K_{CB} + K_{CD}} = 0.6
\end{align*}
\]

Moment Distribution Table

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>FEM</td>
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<td>-16</td>
<td>0</td>
<td>0</td>
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<tr>
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</table>
Shear Conditions

\[ \Sigma H = 0, \quad H_A + H_D + \delta' = 0 \]

\[ H_A = \frac{M_{AB} + M_{AB}}{4}, \quad H_D = \frac{M_{CD} + M_{CD}}{4} \]

\[ \delta' = 11.7 \text{kN} \]

Sway correction factor, \( C = \frac{\delta}{\delta'} = \frac{80}{11.7} = 6.84 \)

Final Moment = Non sway moment + (C x Sway moment)

\[ M_{AB} = 0 + (6.84 \times -12.77) = -87.35 \text{kN-m} \]
\[ M_{BA} = 0 + (6.84 \times -9.68) = -66.21 \text{kN-m} \]
\[ M_{BC} = 0 + (6.84 \times 9.68) = 66.21 \text{kN-m} \]
\[ M_{CB} = 0 + (6.84 \times 12.25) = -83.79 \text{kN-m} \]
\[ M_{CD} = 0 + (6.84 \times -12.25) = -83.79 \text{kN-m} \]
\[ M_{DC} = 0 + (6.84 \times -19.58) = -133.93 \text{kN-m} \]
2. Analyse the frame by moment distribution method.

![Frame diagram]

**Non Sway Analysis**

**Fixed End Moments:**

\[
M_{FAB} = M_{FBA} = 0
\]

\[
M_{FCD} = M_{CD} = 0
\]

\[
M_{FBC} = \frac{-wt^2}{12} = -106.67\text{kN-m}
\]

\[
M_{FCB} = \frac{-wt^2}{12} = 106.67\text{kN-m}
\]

**Stiffness Factor (K)**

\[
K_{BA} = \frac{4EI}{l} = \frac{4EI}{4} = EI
\]

\[
K_{BC} = \frac{4EI}{l} = \frac{4 \times 2EI}{8} = EI
\]

\[
K_{CB} = \frac{4EI}{l} = \frac{4 \times 2EI}{6} = EI
\]

\[
K_{CD} = \frac{3EI}{l} = \frac{3EI}{4} = 0.75EI
\]

**Distribution Factor (Δ)**

\[
Δ_{BA} = \frac{K_{BA}}{K_{BA} + K_{BC}} = 0.5
\]

\[
Δ_{CB} = \frac{K_{CB}}{K_{CB} + K_{CD}} = 0.57
\]

\[
Δ_{BC} = \frac{K_{BC}}{K_{BA} + K_{BC}} = 0.5
\]

\[
Δ_{CD} = \frac{K_{CD}}{K_{CB} + K_{CD}} = 0.43
\]
### Moment Distribution Table

<table>
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<tr>
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<th>A</th>
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<th>C</th>
<th>D</th>
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<td>-1.083</td>
<td>-0.82</td>
</tr>
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<tr>
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<td>36.85</td>
<td>73.76</td>
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<td>61.72</td>
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</table>

Apply the shear conditions to the vertical members

\[ \Sigma_H = 0, \]

\[ H_A + H_D - \delta = 0, \]

\[ H_A = \frac{M_{AB} + M_{AD}}{4}, \quad H_D = \frac{M_{CD} + M_{AD}}{4} \]

\[ \delta = 12.22 \text{kN} \]
Structural Analysis – II

Sway Analysis

Fixed End Moments:

\[ M_{FAB} = M_{FBA} = -\frac{6EI\delta}{l^2} \]
\[ M_{FCD} = M_{FDC} = -\frac{6EI\delta}{l^2} \]

\[ \frac{M_{FAB}}{M_{FCD}} = \frac{-\frac{6EI\delta}{l^2}}{-\frac{6EI\delta}{l^2}} = \frac{-1}{-1} = \frac{-10}{-10} \]

Moment Distribution Table

<table>
<thead>
<tr>
<th></th>
<th>A</th>
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<th>C</th>
<th>D</th>
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<td>-1.43</td>
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<td>Final Moment</td>
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<td>-5.34</td>
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</tbody>
</table>

Shear Conditions

\[ \Sigma H = 0, \quad H_A + H_D + \delta' = 0 \]

\[ H_A = \frac{M_{AB} + M_{AD}}{4}, \quad H_D = \frac{M_{CD} + M_{CD}}{4} \]
δ' = 4.23kN

Sway correction factor,  \( C = \frac{\delta}{\delta'} = \frac{12.22}{4.23} = 2.89 \)

Final Moment = Non sway moment + (C x Sway moment)

\[ M_{AB} = 36.85 + (2.89 \times -7.71) = 14.66 \text{kN-m} \]
\[ M_{BA} = 73.76 + (2.89 \times -5.34) = 58.15 \text{kN-m} \]
\[ M_{BC} = -73.76 + (2.89 \times 5.34) = -58.14 \text{kN-m} \]
\[ M_{CB} = 61.70 + (2.89 \times 3.84) = 72.84 \text{kN-m} \]
\[ M_{CD} = -61.72 + (2.89 \times -3.84) = -72.82 \text{kN-m} \]
\[ M_{DC} = 0 + (2.89 \times 0) = 0 \]
UNIT – 5
KANI’S METHOD

It is a method based on the rotation contributions at each joints. It is one of the oldest and most convenient methods for the analysis of multi storied frames. This method is also an iterative method like moment distribution method.

Procedure
- Calculation of FEM of each span
- Calculation of stiffness factors and rotations factors rotation factor $U = -1/2\left(\frac{K_{BA}}{\sum K_B}\right)$
- Computation of rotation contribution
  \[ M' = U [\Sigma FEM + \Sigma \text{far end contribution}] \]
  The rotation contributions are calculated at intermediate joints by kani’s table. The iteration process is carried out till we get the values almost same as previous cycle.
- Calculation of final support moments
  \[ M_{AB} = M_{FAB} + 2M'_{AB} + M'_{BA} \]
  & \[ M_{BA} = M_{FBA} + 2M'_{BA} + M'_{AB} \]

1. Draw BMD and EC by kani’s method

Fixed End Moments:
\[ M_{FAB} = \frac{30 \times 1.5 \times 8^2}{12} = -160 \text{ kN-m} \]
\[ M_{FBA} = \frac{30 \times 1.5 \times 8^2}{12} = 160 \text{ kN-m} \]
\[ M_{FBC} = \frac{-20 \times 6 \times 4^2}{12^2} = -320 \text{ kN-m} \]
\[ M_{FBD} = \frac{80 \times 6 \times 4^2}{12^2} = 160 \text{ kN-m} \]
\[ M_{FDB} = \frac{-20 \times 6^2}{12} = -60 \text{ kN-m} \]
\[ M_{FAD} = \frac{20 \times 6^2}{12} = 60 \text{ kN-m} \]
Stiffness Factor (K)

\[ K_{BA} = \frac{4EI}{l} = \frac{4 \times 1.5EI}{8} = 0.75EI \]

\[ K_{BC} = \frac{4EI}{l} = \frac{4 \times 2EI}{12} = 0.67EI \]

\[ K_{CB} = \frac{4EI}{l} = \frac{4 \times 2EI}{12} = 0.67EI \]

\[ K_{CD} = \frac{4EI}{l} = \frac{4EI}{6} = 0.67EI \]

Distribution Factor (\( \Delta \))

\[ U_{BA} = -1/2 \left( \frac{K_{BA}}{K_{BA} + K_{BC}} \right) = -1/2 \left( \frac{0.75EI}{0.75EI + 0.67EI} \right) = -0.265 \]

\[ U_{BC} = -1/2 \left( \frac{K_{BC}}{K_{BA} + K_{BC}} \right) = -1/2 \left( \frac{0.67EI}{0.75EI + 0.67EI} \right) = -0.235 \]

\[ U_{CB} = -1/2 \left( \frac{K_{CB}}{K_{CB} + K_{CD}} \right) = -1/2 \left( \frac{0.67EI}{0.67EI + 0.67EI} \right) = -0.25 \]

\[ U_{CD} = -1/2 \left( \frac{K_{CD}}{K_{CB} + K_{CD}} \right) = -1/2 \left( \frac{0.67EI}{0.67EI + 0.67EI} \right) = -0.25 \]

<table>
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<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
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<td>-160</td>
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<td>-320</td>
<td>-60</td>
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<td>60</td>
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<tr>
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<td>-36.42</td>
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<tr>
<td>52.05</td>
<td>46.16</td>
<td>-36.54</td>
<td>-36.54</td>
</tr>
<tr>
<td>52.08</td>
<td>46.19</td>
<td>-36.55</td>
<td>-36.55</td>
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<tr>
<td>52.08</td>
<td>46.19</td>
<td>-36.55</td>
<td>-36.55</td>
</tr>
</tbody>
</table>

\[ M'_{BA} = 52.08 \quad M'_{BC} = 46.19 \quad M'_{CB} = -36.55 \quad M'_{CD} = -36.55 \]

Final Moments

\[ M_{AB} = M_{FAB} + 2 M'_{AB} + M'_{BA} \]

\[ M_{AB} = -160 + 2 \times 0 + 52.08 = -107.92 \text{kN-m} \]

\[ M_{BA} = 160 + 2 \times (52.08) + 0 = 264.16 \text{kN-m} \]
\[ M_{BC} = -320 + 2 \times (46.19) - 36.42 = -264.17 \text{kN-m} \]

\[ M_{CB} = 160 + 2 \times (-36.42) + 46.19 = 133.1 \text{kN-m} \]

\[ M_{CD} = -60 + 2 \times (-36.55) - 0 = -133.1 \text{kN-m} \]

\[ M_{DC} = 60 + 2 \times (0) - 36.55 = 23.45 \text{kN-m} \]

2. Draw BMD and EC by Kani’s method

\[ M_{FAB} = \frac{-w l^2}{12} = \frac{-30 \times 1.5^2}{12} = -4.125 \text{kN-m} \]

\[ M_{FBA} = \frac{w l^2}{12} = \frac{30 \times 8^2}{12} = 160 \text{kN-m} \]

\[ M_{FBC} = \frac{-w a b^2}{l^2} = \frac{-100 \times 4 \times 8^2}{12^2} = -320 \text{kN-m} \]

\[ M_{FCB} = \frac{w b a^2}{l^2} = \frac{80 \times 8 \times 4^2}{12^2} = 160 \text{kN-m} \]

\[ M_{FCD} = \frac{-w l^2}{12} = \frac{-20 \times 6^2}{12} = -60 \text{kN-m} \]

\[ M_{FDC} = \frac{w l^2}{12} = \frac{26 \times 6^2}{12} = 60 \text{kN-m} \]

\[ M_{FCD} = -90 \text{kN-m} \]

\[ M_{FDC} = 0 \text{kN-m} \]
Stiffness Factor (K)

\[ K_{BA} = \frac{4EI}{l} = \frac{4 \times 1.5EI}{8} = 0.75EI \]

\[ K_{BC} = \frac{4EI}{l} = \frac{4 \times 2EI}{12} = 0.67EI \]

\[ K_{CB} = \frac{4EI}{l} = \frac{4 \times 2EI}{12} = 0.67EI \]

\[ K_{CD} = \frac{3EI}{l} = \frac{3 EI}{6} = 0.5EI \]

Distribution Factor (Δ)

\[ U_{BA} = -1/2 \left[ \frac{K_{BA}}{K_{BA} + K_{BC}} \right] = -1/2 \left[ \frac{0.75EI}{0.75EI + 0.67EI} \right] = -0.265 \]

\[ U_{BC} = -1/2 \left[ \frac{K_{BC}}{K_{BA} + K_{BC}} \right] = -1/2 \left[ \frac{0.67EI}{0.75EI + 0.67EI} \right] = -0.235 \]

\[ U_{CB} = -1/2 \left[ \frac{K_{CB}}{K_{CB} + K_{CD}} \right] = -1/2 \left[ \frac{0.67EI}{0.67EI + 0.5EI} \right] = -0.286 \]

\[ U_{CD} = -1/2 \left[ \frac{K_{CD}}{K_{CB} + K_{CD}} \right] = -1/2 \left[ \frac{0.5EI}{0.67EI + 0.5EI} \right] = -0.214 \]

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<td>-160</td>
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<tr>
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</table>

\[ M'_{BA} = 51.1 \quad M'_{BC} = 45.35 \quad M'_{CB} = -32.99 \quad M'_{CD} = -24.68 \]

Final Moments

\[ M_{AB} = M_{FAB} + 2M'_{AB} + M'_{BA} \]

\[ M_{AB} = -160 + 2 \times (0) + 51.1 = -108.9 \text{kN-m} \]

\[ M_{BA} = 160 + 2 \times (51.1) + 0 = 262.28 \text{kN-m} \]

\[ M_{BC} = -320 + 2 \times (45.35) - 32.99 = -262.29 \text{kN-m} \]
\[ M_{CB} = 160 + 2 \times (-32.99) + 45.35 = 139.36 \text{kN-m} \]

\[ M_{CD} = -90 + 2 \times (-24.68) - 0 = -139.36 \text{kN-m} \]

\[ M_{DC} = 0 \]

3. Draw BMD and EC by kani’s method

Fixed End Moments:
\[ M_{FAB} = \frac{-wa l^2}{12} = \frac{-80 \times 8 \times 8^2}{12^2} = -160 \text{kN-m} \]
\[ M_{FBA} = \frac{wa l^2}{12} = \frac{30 \times 8 \times 8^2}{12} = 160 \text{kN-m} \]
\[ M_{FBC} = \frac{-wa l^2}{12} = \frac{-180 \times 8 \times 8^2}{12^2} = -320 \text{kN-m} \]
\[ M_{FCB} = \frac{wb \cdot a^2}{l^2} = \frac{80 \times 8 \times 4^2}{12^2} = 160 \text{kN-m} \]
\[ M_{FCD} = M_{DC} = 0 \]
\[ M_{FBC} = \frac{-wa l^2}{12} = \frac{-180 \times 8 \times 8^2}{12^2} = -320 \text{kN-m} \]
\[ M_{FCB} = \frac{wb \cdot a^2}{l^2} = \frac{80 \times 8 \times 4^2}{12^2} = 160 \text{kN-m} \]

\[ = 100 \text{kN-m} \]

\[ M_{FCD} = -220 \text{kN-m} \]

\[ M_{FCB} = 360 \text{kN-m} \]
Structural Analysis – II

Stiffness Factor \(K\)

\[
K_{BA} = \frac{4EI}{l} = \frac{4 \times 1.5EI}{8} = 0.75EI
\]

\[
K_{BC} = \frac{3EI}{l} = \frac{3 \times 2EI}{12} = 0.5EI
\]

\[
K_{CB} = \frac{4EI}{l} = \frac{4 \times 2EI}{12} = 0.67EI
\]

Distribution Factor \(U\)

\[
U_{BA} = -\frac{1}{2} \left[ \frac{K_{BA}}{K_{BA} + K_{BC}} \right] = -\frac{1}{2} \left[ \frac{0.75EI}{0.75EI + 0.5EI} \right] = -0.3
\]

\[
U_{BC} = -\frac{1}{2} \left[ \frac{K_{BC}}{K_{BA} + K_{BC}} \right] = -\frac{1}{2} \left[ \frac{0.5EI}{0.75EI + 0.5EI} \right] = -0.2
\]

\[
U_{CB} = -\frac{1}{2} \left[ \frac{K_{CB}}{K_{CB} + K_{CD}} \right] = -\frac{1}{2} \left[ \frac{0.67EI}{0.67EI + 0.5EI} \right] = -0.5
\]

Final Moments

\[
M_{AB} = M_{FAB} + 2M'_{AB} + M'_{BA}
\]

\[
M_{AB} = -160 + 2 \times (0) + 18 = -142kN-m
\]

\[
M_{BA} = 160 + 2 \times (18) + 0 = 196kN-m
\]

\[
M_{BC} = -220 + 2 \times (12) + 0 = -196kN-m
\]

\[
M_{CB} = 360kN-m
\]

\[
M_{CD} = -360kN-m
\]
Sinking of Support

1. Analyse the continuous beam by Kani’s method, support B sinks by 10mm. Support A rotates by 0.002rad. Draw the BMD and EC. Given $E = 200 \times 10^6$ kPa and $I = 100 \times 10^6$ mm$^4$

**Fixed End Moments:**

$M_{FAB} = \frac{-wa b^2}{t^2} + \frac{4EI\delta}{t^2} - \frac{6EI\delta}{t^2} = -\frac{40 \times 2 \times 2^2}{4^2} + \frac{4 \times 2 \times 10^4 \times 0.002}{4} - \frac{6 \times 2 \times 1 \times 0.001}{4} = -55$ kN-m

$M_{FBA} = \frac{-wb a^2}{t^2} + \frac{2EI\delta}{t^2} = -\frac{10 \times 2 \times 2^2}{4^2} + \frac{2 \times 2 \times 10^4 \times 0.002}{4} = -35$ kN-m

$M_{FBC} = \frac{-w a b^2}{12} - \frac{6EI(-\delta)}{t^2} = \frac{-180 \times 4 \times 8^2}{12^2} + \frac{6 \times 2 \times 10^4 \times 0.001}{6^2} = 3.33$ kN-m

$M_{FCB} = \frac{M_{FCB}}{t^2} = \frac{63.33}{6^2} = 63.33$ kN-m

$M_{FCD} = M_{CD} = -20 \times 2 \times 1 = -40$ kN-m

$M_{FBC} = \frac{-wa b^2}{12} = \frac{-180 \times 4 \times 8^2}{12^2} = 3.33$ kN-m

$M_{FCB} = \frac{wb a^2}{12} = -\frac{80 \times 8 \times 4^2}{12^2} = 63.33$ kN-m

$M_{FCD} = -3.335$ kN-m

$M_{FCB} = 40$ kN-m

**Stiffness Factor (K)**

$K_{BA} = \frac{4EI}{l} = \frac{4 \times EI}{4} = EI$

$K_{BC} = \frac{3EI}{l} = \frac{3 \times EI}{6} = 0.5EI$
\[ K_{CB} = \frac{4EI}{l} = \frac{4 \times EI}{6} = 0.67EI \]

**Distribution Factor ( )**

\[ U_{BA} = -\frac{1}{2} \left[ \frac{K_{BA}}{K_{BA} + K_{BC}} \right] = -\frac{1}{2} \left[ \frac{EI}{EI + 0.5EI} \right] = -0.33 \]

\[ U_{BC} = -\frac{1}{2} \left[ \frac{K_{BC}}{K_{BA} + K_{BC}} \right] = -\frac{1}{2} \left[ \frac{0.5EI}{EI + 0.5EI} \right] = -0.17 \]

\[ U_{CB} = -\frac{1}{2} \left[ \frac{K_{CB}}{K_{CB} + K_{CD}} \right] = -\frac{1}{2} \left[ \frac{0.67EI}{0.67EI + 0} \right] = -0.5 \]

\[ \begin{array}{c|c|c|c|c}
\hline
 & A & B & C & D \\
\hline
M'_{BA} & -55 & -35 & -38.35 & -40 \\
12.66 & & & & 0 \\
\hline
M'_{BC} & -3.35 & 40 & & -40 \\
6.4 & & 0 & & \\
\hline
\end{array} \]

\[ M'_{BA} = 12.66 \quad M'_{BC} = 6.4 \]

**Final Moments**

\[ M_{AB} = M_{FAB} + 2 M'_{AB} + M'_{BA} \]

\[ M_{AB} = -55 + 2 \times (0) + 12.66 = -42.34 \text{kN-m} \]

\[ M_{BA} = -35 + 2 \times (12.66) + 0 = -9.68 \text{kN-m} \]

\[ M_{BC} = -3.33 + 2 \times (6.4) + 0 = 9.47 \text{kN-m} \]

\[ M_{CB} = 40 \text{kN-m} \]

\[ M_{CD} = -40 \text{kN-m} \]
Analysis of Non-Sway Frames

1. Draw the BMD and EC by Kani’s method

![Frame Diagram]

**Fixed End Moments:**

\[
M_{FAB} = \frac{-wL^2}{6} \cdot \frac{a^2}{L^2} = \frac{-20 \times 1 \times 3^2}{4^2} = -22.5 \text{ kN-m}
\]

\[
M_{FBA} = \frac{wL^2}{6} \cdot \frac{a^2}{L^2} = \frac{20 \times 1 \times 3^2}{4^2} = 22.5 \text{ kN-m}
\]

\[
M_{FBD} = \frac{-wL^2}{6} \cdot \frac{a^2}{L^2} = \frac{-20 \times 3 \times 2^2}{5^2} = 9.6 \text{ kN-m}
\]

\[
M_{FDB} = \frac{wL^2}{6} \cdot \frac{a^2}{L^2} = \frac{20 \times 2 \times 3^2}{5^2} = -14.4 \text{ kN-m}
\]

\[
M_{FBC} = M_{BC} = -10 \times 4 \times 2 = -80 \text{ kN-m}
\]

**Stiffness Factor (K)**

\[
K_{BA} = \frac{4EI}{l^5} = \frac{4 \times 1.5EI}{8} = 0.75EI
\]

\[
K_{BC} = 0
\]

\[
K_{BD} = \frac{4EI}{l^5} = \frac{4EI}{5} = 0.8EI
\]

**Distribution Factor (Δ)**

\[
U_{BA} = -1/2 \left[ \frac{K_{BA}}{K_{BA} + K_{BC}} \right] = -1/2 \left[ \frac{EI}{EI + 0.8EI} \right] = -0.278
\]

\[
U_{BC} = 0
\]

\[
U_{CB} = -1/2 \left[ \frac{K_{BC}}{K_{BC} + K_{CD}} \right] = -1/2 \left[ \frac{0.8EI}{0.8EI + EI} \right] = -0.22
\]
Final Moments

\[ M_{AB} = M_{FAB} + 2M'_{AB} + M'_{BA} \]
\[ M_{AB} = -22.5 + 2 \times 0 + 13.32 = -9.18 \text{kN-m} \]
\[ M_{BA} = 22.5 + 2 \times 13.32 + 0 = 49.14 \text{kN-m} \]
\[ M_{BC} = -80 \]
\[ M_{BD} = 9.6 + 2 \times 10.54 + 0 = 30.68 \text{kN-m} \]
\[ M_{DB} = -14.4 + 2 \times 0 + 10.54 = -3.86 \text{kN-m} \]
UNIT-6

FLEXIBILITY MATRIX METHOD

The systematic development of consistent deformation method in the matrix form has lead to flexibility matrix method. The method is also called force method. Since the basic unknowns are the redundant forces in the structure.

This method is exactly opposite to stiffness matrix method.
The flexibility matrix equation is given by

\[
[P] [F] = \{[\Delta] - [\Delta_L]\}
\]

\[
[P] = [F]^{-1}\{[\Delta] - [\Delta_L]\}
\]

Where,
- \([P]\) = Redundant in matrix form
- \([F]\) = Flexibility matrix
- \([\Delta]\) = Displacement at supports
- \([\Delta_L]\) = Displacement due to load

1. Analyse the continuous beam shown in the figure by flexibility matrix method, draw BMD

![Continuous Beam Diagram](image)

Static Indeterminacy SI = 2 (M_A and M_B)
M_A and M_B are the redundant
Let us remove the redundant to get primary determinate structure

![Modified Beam Diagram](image)
\[
\begin{bmatrix}
1L \\
2L
\end{bmatrix} = \begin{bmatrix}
\Delta_1L \\
\Delta_2L
\end{bmatrix}
\]

$1L = \text{Rotation at } A = \text{SF at } A'$

$1L = \frac{1}{2} \left[ \frac{2}{3} \times 4 \times \frac{120}{EI} \right]$

$1L = \frac{160}{EI}$

$2L = \text{Rotation at } A = \text{SF at } B'$

$2L = \frac{216.25}{EI}$

$\begin{bmatrix}
\frac{1}{EI} \\
\frac{160}{EI} \\
\frac{216.25}{EI}
\end{bmatrix}$

Note: The rotation due to sagging is taken as positive. The moments producing due to sagging are also taken as positive.

To get Flexibility Matrix

Apply unit moment to joint A

\[
[F] = \begin{bmatrix}
\delta_{11} & \delta_{12} \\
\delta_{21} & \delta_{22}
\end{bmatrix}
\]
\[ \delta_{11} = \frac{ml}{3EI} = \frac{1 \times 4}{3EI} = \frac{1.33}{EI} \]

\[ \delta_{21} = \frac{ml}{6EI} = \frac{1 \times 4}{6EI} = \frac{0.67}{EI} \]

Apply unit moment to joint A

\[ \delta_{12} = \frac{ml}{6EI} = \frac{1 \times 4}{6EI} = \frac{0.67}{EI} \]

\[ \delta_{22} = \frac{ml}{3EI} + \frac{ml}{3EI} = \frac{1 \times 4}{3EI} + \frac{1 \times 3}{EI} = 2.33 \]

\[
[F] = \begin{pmatrix}
\delta_{11} & \delta_{12} \\
\delta_{21} & \delta_{22}
\end{pmatrix} = \frac{1}{EI} \begin{pmatrix}
1.33 & 0.67 \\
0.67 & 1.33
\end{pmatrix}
\]

Apply the flexibility equation

\[ [P] = [F]^{-1} \cdot \left[ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} - [L] \right] \]

\[ [L] = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \]

\[ [P] = EI \begin{pmatrix}
1.33 & 0.67 \\
0.67 & 1.33
\end{pmatrix}^{-1} \left\{ \begin{bmatrix} 0 \\ -160 \end{bmatrix} \right\} \]

\[ [P] = EI \begin{pmatrix}
1.33 & 0.67 \\
0.67 & 1.33
\end{pmatrix}^{-1} \left\{ \begin{bmatrix} 0 \\ -160 \end{bmatrix} \right\} \]

\[ [P] = \begin{pmatrix}
M_{AB} \\
M_{BA}
\end{pmatrix} = \begin{pmatrix}
-86.00 \\
-68.08
\end{pmatrix} \text{kN-m} \]
2. Analyse the continuous beam shown in the figure by flexibility matrix method, draw BMD

Static Indeterminacy $SI = 2$ ($M_B$ and $M_C$)

$M_B$ and $M_C$ are the redundant

Let us remove the redundant to get primary determinate structure
\[ \Delta L = \begin{pmatrix} 1L \\ 2L \end{pmatrix} \]

1L = Rotation at B = SF at B'

\[ = V_{B1}' + V_{B2}' \]

\[ 1L = \frac{3946.67}{EI} \]

2L = Rotation at C = SF at C'

\[ = V_{C1}' + V_{C2}' \]

\[ 2L = \frac{2293.33}{EI} \]

\[ \frac{1}{EI} \begin{pmatrix} 3946.67 \\ 2293.33 \end{pmatrix} \]
To get Flexibility Matrix

Apply unit moment to joint A

\[
[F] = \begin{pmatrix}
\delta_{11} & \delta_{12} \\
\delta_{21} & \delta_{22}
\end{pmatrix}
\]

\[
\delta_{11} = \frac{ml}{3EI} + \frac{ml}{3EI} = \frac{1 \times 12}{3EI} + \frac{1 \times 12}{3EI} = \frac{8}{EI}
\]

\[
\delta_{21} = \frac{ml}{6EI} = \frac{1 \times 12}{6EI} = \frac{2}{EI}
\]

Apply unit moment to joint A

\[
\delta_{12} = \frac{ml}{6EI} = \frac{1 \times 12}{6EI} = \frac{2}{EI}
\]

\[
\delta_{22} = \frac{ml}{3EI} + \frac{ml}{3EI} = \frac{1 \times 12}{3EI} + \frac{1 \times 12}{3EI} = \frac{8}{EI}
\]

\[
[F] = \begin{pmatrix}
\delta_{11} & \delta_{12} \\
\delta_{21} & \delta_{22}
\end{pmatrix} = \frac{1}{EI} \begin{pmatrix} 8 & 2 \\ 2 & 8 \end{pmatrix}
\]

Apply the flexibility equation

\[
[P] = [F]^{-1} \{ [ ] - [ L] \}
\]

\[
[ L] = \begin{pmatrix} 0 \\ 0 \end{pmatrix}
\]
SINKING OF SUPPORT

1. Analyse the continuous beam by flexibility method, support B sinks by 5mm. Sketch the BMD and EC given EI = 15 X 10^3 kN-m^2.

NOTE: In this case of example with sinking of supports, the redundant should be selected as the vertical reaction. Static indeterminacy is equal to 2. Let V_B and V_C be the redundant, remove the redundant to get the primary structure.
\[ [\Delta L] = \begin{pmatrix} 1L \\ 2L \end{pmatrix} \]

1L = Displacement at B in primary determinate structure = BM at B’ in conjugate beam

\[ 1L = \left[ \frac{1}{2} \times 6 \times \frac{360}{EI} \times (2/3 \times 6) \right] + \left[ \frac{1}{3} \times 6 \times \frac{270}{EI} \times (3/4 \times 6) \right] \]

\[ 1L = \frac{8910}{EI} \]

2L = Displacement at C in primary determinate structure = BM at C’ in conjugate beam

\[ 2L = \left[ \frac{1}{2} \times 6 \times \frac{360}{EI} \times (2/3 \times 6 + 4) \right] + \left[ \frac{1}{3} \times 6 \times \frac{270}{EI} \times (3/4 \times 6 + 4) \right] \]

\[ 2L = \frac{19070}{EI} \]

\[ [\Delta L] = \begin{pmatrix} \frac{8910}{EI} \\ \frac{19070}{EI} \end{pmatrix} \]

To get Flexibility Matrix

Apply unit Load at B

A

6m

B

1kN

C

3/EI

A’

B’

C’
\[
[F] = \begin{pmatrix}
\delta_{11} & \delta_{12} \\
\delta_{21} & \delta_{22}
\end{pmatrix}
\]

\[
\delta_{11} = -\frac{1}{2} \times 6 \times \frac{3}{EI} \times (2/3 \times 6) = \frac{-36}{EI}
\]

\[
\delta_{21} = -\frac{1}{2} \times 6 \times \frac{3}{EI} \times (2/3 \times 6 \times 4) = \frac{-72}{EI}
\]

Apply unit load at C

Apply the flexibility equation

\[
[P] = [F]^{-1} \left( \begin{pmatrix} 0.005 \\ 0 \end{pmatrix} \right)
\]
\[ [P] = EI \begin{pmatrix} -36 & -72 \\ -72 & 177.33 \end{pmatrix}^{-1} \begin{pmatrix} 0.005 \\ 0 \end{pmatrix} \frac{1}{EI} \begin{pmatrix} 8910 \\ 19070 \end{pmatrix} \]

\[ [P] = \begin{pmatrix} V_B \\ V_C \end{pmatrix} = \begin{pmatrix} 161.43 \\ 41.98 \end{pmatrix} \text{kN-m} \]

Support Reaction

\[ V_A = 96.64 \text{kN}, \quad V_{B1} = 83.36 \text{kN}, \quad V_{B2} = 78.07 \text{kN}, \quad V_C = 41.98 \text{kN} \]

\[ V_B = V_{B1} + V_{B2} = 161.43 \text{kN} \]

\[ \begin{pmatrix} M_A \\ M_B \end{pmatrix} = \begin{pmatrix} 112.48 \\ 72.28 \end{pmatrix} \text{kN-m} \]
UNIT-7
STIFFNESS MATRIX METHOD

The systematic development of slope deflection method in the matrix form has lead to Stiffness matrix method. The method is also called Displacement method. Since the basic unknowns are the displacement at the joint.

The stiffness matrix equation is given by

\[
[\Delta] = [K]^{-1} ([P] - [P_L])
\]

Where,
- \([P]\) = Redundant in matrix form
- \([F]\) = Stiffness matrix
- \([P]\) = Final force at the joints in matrix form
- \([P_L]\) = force at the joints due to applied load in matrix form

1. Analyse the continuous beam by Stiffness method Sketch the BMD

![Diagram of continuous beam with forces and displacements]

Kinematic Indeterminacy \( KI = 1 (\theta_B) \)

\([P_L]\) = \( M_{FBA} \cdot M_{FBC} \)
\[
= \frac{\omega t^2}{12} + \left( -\frac{\omega t}{8} \right) = \frac{20 \times 6^2}{12} - \frac{120 \times 6}{8}
\]

\([P_L]\) = -30kN-m

Apply unit displacement at joint B.

\[
[K] = \frac{4EIL_0}{l} + \frac{4EIL_0}{l} = \frac{4EI}{6} + \frac{4EI}{6} = 1.33EI \quad (\theta=1)
\]
By condition of equilibrium at joint B

\[ [P] = 0 \]

\[ [\Delta] = [K]^{-1} ([P] - [P_L]) \]

\[ \theta_B = \frac{1}{133EI} \{ [0] - [-30] \} = \frac{22.56}{EI} \]

Slope deflection equation

\[ M_{AB} = M_{FAB} + \frac{2EI}{l} (2\theta_A + \theta_C) \]

\[ = -60 + \frac{2EI}{6} (2\theta_A + \frac{22.5}{EI}) \]

\[ (\theta_A = 0 \text{ due to fixity at support A}) \]

\[ M_{AB} = -52.5 \text{kN-m} \]

\[ M_{BA} = M_{FBA} + \frac{2EI}{l} (2\theta_B + \theta_A) \]

\[ = 60 + \frac{2EI}{6} (2\theta_B + \frac{22.5}{EI}) \]

\[ M_{BA} = 75.04 \text{kN-m} \]

\[ M_{BC} = M_{FBC} + \frac{2EI}{l} (2\theta_B + \theta_C) \]

\[ = -90 + \frac{2EI}{6} (2\theta_B + \frac{22.5}{EI} + 0) \]

\[ M_{BC} = -75 \text{kN-m} \]

\[ M_{CB} = M_{FCB} + \frac{2EI}{l} (2\theta_C + \theta_B) \]

\[ = 90 + \frac{2EI}{6} (0 + \frac{22.5}{EI}) \]

\[ M_{CB} = 97.52 \text{kN-m} \]
2. Analyse the continuous beam by Stiffness method Sketch the BMD

Kinematic Indeterminacy \( K_I = 2 (\theta_B & \theta_C) \)

\[ [P_{1L}] = M_{FBA} + M_{FBC} \]
\[ = \frac{wl^2}{12} + \left( -\frac{wl}{6} \right) = \frac{60 \times 4^2}{12} - \frac{100 \times 3}{6} = 42.5\text{kN-m} \]

\[ [P_{2L}] = M_{FCB} = \frac{wl^3}{24} = \frac{100 \times 3^3}{24} = 37.5\text{kN-m} \]

\[ [P_L] = \begin{pmatrix} P_{1L} \\ P_{2L} \end{pmatrix} = \begin{pmatrix} 42.5 \\ 37.5 \end{pmatrix} \text{kN-m} \]

Apply unit displacement at joint B.

\[ K_{11} = \frac{4EI\theta}{l} + \frac{4EI\theta}{l} - \frac{4EI}{4} + \frac{4EI}{3} = 2.33EI \quad (\theta = 1) \]

\[ K_{21} = \frac{2EI\theta}{l} = \frac{2EI}{3} = 0.67EI \]

Apply unit displacement at joint B.

\[ K_{12} = \frac{2EI\theta}{l} = \frac{2EI}{3} = 0.67EI \]
\[ K_{22} = \frac{4EI\theta}{l} = \frac{4EI}{3} = 1.33EI \]
By condition of equilibrium at joint B

\[ [P] = 0 \]
\[ [\Delta] = [K]^{-1}([P] - [P_L]) \]

\[ \begin{pmatrix} 2.33 & 0.67 \\ 0.67 & 1.33 \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ 42.5 \end{pmatrix} = \begin{pmatrix} 0 \\ 37.5 \end{pmatrix} \]

\[
\begin{pmatrix} \theta_B \\ \theta_C \end{pmatrix} = \frac{1}{EI} \begin{pmatrix} -11.88 \\ -22.19 \end{pmatrix}
\]

Slope deflection equation

\[ M_{AB} = M_{FAB} + \frac{2EI}{l}(2\theta_A + \theta_B) \]
\[ = -80 + \frac{2EI}{4} \left(2\theta_A - \frac{11.88}{EI} \right) \]
\[ (\theta_A = 0 \text{ due to fixity at support A}) \]

\[ M_{AB} = -85.94 \text{kN-m} \]

\[ M_{BA} = M_{FBA} + \frac{2EI}{l}(2\theta_B + \theta_A) \]
\[ = 80 + \frac{2EI}{4} \left(2X \frac{-11.88}{EI} + \theta_A \right) \]

\[ M_{BA} = 68.12 \text{kN-m} \]

\[ M_{BC} = M_{FBC} + \frac{2EI}{l}(2\theta_B + \theta_C) \]
\[ = -37.5 + \frac{2EI}{6} \left(2X \frac{-11.88}{EI} + \frac{-22.9}{EI} \right) \]

\[ M_{BC} = -68.6 \text{kN-m} \]

\[ M_{CB} = 0 \]
**Sinking of support**

1. Analyse the continuous beam shown in figure by stiffness method. Support B sinks by 300/EI units and support C sinks by 200/EI units.

Kinematic Indeterminacy $K_I = 2 (\theta_B & \theta_C)$

\[ \begin{align*}
[P_{1L}] &= M_{FBA} + M_{FBC} - \frac{6EI\delta}{l^2} \\
&= \frac{100 \times 8}{8} - \frac{60 \times 8}{8} + \frac{6 \times 300}{8^2} - \frac{6 \times 100}{8^2} = 21.25 \text{kN-m} \\
[P_{2L}] &= M_{FCB} - \frac{6EI\delta}{l^2} = \frac{60 \times 8}{8} + \frac{6 \times 100}{8^2} = 69.38 \text{kN-m}
\end{align*} \]

Apply unit displacement at joint B.

\[ K_{11} = \frac{4EI\theta}{l} + \frac{4EI\theta}{l} - \frac{4EI\theta}{l} = 0.25EI \] \(\theta = 1\) (0=1)

\[ K_{21} = \frac{2EI\theta}{8} = \frac{2EI}{3} = 0.25EI \]

Apply unit displacement at joint B.

\[ K_{12} = \frac{2EI\theta}{l} = \frac{2EI}{8} = 0.25EI \]
\[ K_{22} = \frac{4EI_B}{l} = \frac{4EI}{8} = 0.50EI \]

By condition of equilibrium at joint B

\[ [P] = 0 \]

\[ [\Delta] = [K]^{-1}[[P] - [P_L]] \]

\[
\begin{bmatrix}
1 & 0.25 \\
0.25 & 0.50
\end{bmatrix}
\begin{bmatrix}
0 \\
0
\end{bmatrix}
= \begin{bmatrix}
21.25 \\
69.38
\end{bmatrix}
\]

\[
\begin{bmatrix}
\theta_B \\
\theta_C
\end{bmatrix}
= \frac{1}{EI} \begin{bmatrix}
15.36 \\
-146.44
\end{bmatrix}
\]

Slope deflection equation

\[ M_{AB} = M_{FAB} + \frac{2EI}{l} (2\theta_A + \theta_B - \frac{3\delta}{l}) \]

\[ = -100 + \frac{2EI}{8} \left(2\theta_A + \frac{15.36}{EI} - \frac{3EI(300/ET)}{8}\right) \]

\[ (\theta_A = 0 \text{ due to fixity at support A}) \]

\[ M_{AB} = -124.29 \text{kN-m} \]

\[ M_{BA} = M_{FBA} + \frac{2EI}{l} (2\theta_A + \theta_B - \frac{3\delta}{l}) \]

\[ = 100 + \frac{2EI}{8} \left(\theta_A + 2X \frac{15.36}{EI} - \frac{3EI(300/ET)}{8}\right) \]

\[ M_{BA} = 79.55 \text{kN-m} \]

\[ M_{BC} = M_{FBC} + \frac{2EI}{l} (2\theta_B + \theta_C - \frac{3\delta}{l}) \]

\[ = -60 + \frac{2EI}{8} \left(2X \frac{15.36}{EI} + \frac{-146.44}{EI} - \frac{3EI(-100/ET)}{5}\right) \]

\[ M_{BC} = -79.55 \text{kN-m} \]

\[ M_{CB} = 0 \]
Analysis of frames

1. Analyse the frame by stiffness method

\[ [P_{1L}] = M_{FBA} - M_{FBC} + M_{FCD} \]

\[ = \frac{100 \times 2 \times 3^2}{12} - \frac{30 \times 3^2}{12} = 25.5 \text{kN-m} \]

\[ [P_{2L}] = M_{FCB} = \frac{30 \times 3^2}{12} = 22.5 \text{kN-m} \]

\[ [P_L] = \begin{pmatrix} P_{1L} \\ P_{2L} \end{pmatrix} = \begin{pmatrix} 25.5 \\ 22.5 \end{pmatrix} \text{kN-m} \]
Apply unit displacement at joint B.

\[
K_{11} = \frac{4EI\theta}{l} + \frac{4EI\theta}{l} + \frac{4EI\theta}{l} + \frac{4 \times 2EI}{3} + \frac{4EI}{3} = 4.267EI \quad (\theta=1)
\]

\[
K_{21} = \frac{2EI\theta}{l} = \frac{2EI}{3} = 0.67EI
\]

Apply unit displacement at joint C.

\[
K_{12} = \frac{2EI\theta}{l} = \frac{2EI}{3} = 0.67EI
\]

\[
K_{22} = \frac{4EI\theta}{l} = \frac{4EI}{3} = 1.33EI
\]

By condition of equilibrium at joint B

\[
[P] = 0
\]

\[
[ ] = [K]^{-1}\{[P] - [P_L]\} 
\]
\[
\left[ \begin{array}{cc}
4.267 & 0.67 \\
0.67 & 1.33
\end{array} \right]^{-1} \left\{ \begin{array}{c}
0 \\
0
\end{array} \right\} - \left\{ \begin{array}{c}
25.5 \\
22.5
\end{array} \right\}
\]

\[
\left. \begin{array}{c}
\theta_B \\
\theta_C
\end{array} \right] = \frac{1}{EI} \left[ \begin{array}{c}
-3.604 \\
-15.01
\end{array} \right]
\]

Slope deflection equation

\[M_{AB} = M_{FAB} + \frac{2EI}{l} (2\theta_A + \theta_B)\]
\[= -72 + \frac{2 \times 2EI}{5} \left( 2\theta_A + \frac{-3.604}{EI} \right)\]
\[(\theta_A = 0 \text{ due to fixity at support A})\]
\[M_{AB} = -74.88 \text{kN-m}\]

\[M_{BA} = M_{FBA} + \frac{2EI}{l} (2\theta_A + \theta_B)\]
\[= 72 + \frac{2 \times 2EI}{5} \left( \theta_A + 2 \times \frac{-3.604}{EI} \right)\]
\[M_{BA} = 42.23 \text{kN-m}\]

\[M_{BC} = M_{FBC} + \frac{2EI}{l} (2\theta_B + \theta_C)\]
\[= -22.5 + \frac{2EI}{3} \left( 2 \times \frac{-3.604}{EI} + \frac{-15.1}{EI} \right)\]
\[M_{BC} = -37.37 \text{kN-m}\]

\[M_{BD} = M_{FBD} + \frac{2EI}{l} (2\theta_B + \theta_D)\]
\[= 0 + \frac{2EI}{3} \left( 2 \times \frac{-3.604}{EI} + 0 \right)\]
\[M_{BD} = -4.81 \text{kN-m}\]

\[M_{DB} = M_{FDB} + \frac{2EI}{l} (2\theta_D + \theta_B)\]
\[= 0 + \frac{2EI}{3} \left( 2 \times 0 + \frac{-3.604}{EI} \right)\]
\[M_{DB} = -2.402 \text{kN-m}\]

\[M_{CB} = 0\]
UNIT – 8
STRUCTURAL DYNAMICS

Terminology:-

1. Periodic motion: - Periodic motion is a motion that repeats itself at equal interval of time.

![Vibration Parameter vs Time Graph]

2. Time Periodic: - The time required for motion of one cycle is called time period.

![Vibration Parameter vs Time Graph]

3. Cycle: - The motion during one time period is called cycle.

![Vibration Parameter vs Time Graph]

4. Frequency: - Frequency is a no of cycle per unit time.

![Vibration Parameter vs Time Graph]

5. Amplitude (A): - Peak displacement from the mean position is called amplitude.

![Vibration Parameter vs Time Graph]
6. **Free Vibration**: - It is the vibration under no external vibration but occurs with the initial displacement.

7. **Natural frequency \( (f_n) \)**: - It is the frequency of a body under free vibration.
   
   Circular Natural frequency \( (w_n) \): 
   \[
   w_n = 2\pi f_n
   \]

8. **Damping**: - It is the interval resistance to the motion of a body.

9. **Degree of freedom**: - It is the no of independent co-ordinate required to determine (define) the displacement configuration at any time is called the degree of freedom.

---

**Simple harmonic Motion**: -

- Is a periodic reciprocating motion is accelerate always directed towards mean position and is proportion to the displacement from mean position.

**Vector representation of SHM**: -
Let us consider a displacement OP as \( y = A \sin \omega_n t \)

Where,
\[ y = \text{displacement vector} \]
\[ A = \text{Amplitude} \]
\[ \omega_n = \text{natural circular frequency} \]

Differentiate 1 w.r.t ‘t’
\[
Y_1 = A \omega_n \sin \omega_n t
\]
\[
= A \omega_n \left[ \sin(\omega_n t + \pi/2) \right]
\]

Differentiate 2 w.r.t ‘t’
\[
Y_2 = -A \omega_n^2 \sin \omega_n t
\]
\[
= A \omega_n^2 \left[ \sin(\omega_n t + \pi) \right]
\]

**Characteristic**
1. Velocity vector leads the displacement vector by \( \pi/2 \)
2. Displacement, velocity and acceleration vector will rotate in counter clockwise direction with same frequency

**Vibration analysis**
1. Mathematical modelling
2. Formulation of governing equation of motion
3. Solution of equations of motion
4. Analysis and interpretation of results

**Model** :- A Mathematical representation of a physical system/a physical process

![Physical System and Model Diagram](image-url)
Differential equation of motion by Simple harmonic equation

Approach:- consider a single degree of freedom spring mass system as shown in figure which oscillates about the mean position as shown in the figure.

Let the \( y(t) = A \sin \omega_n t \)

The velocity is

\[
Y_1(t) = A \omega_n \cos \omega_n t \\
= A \omega_n \sin(\omega_n t + \pi/2)
\]

The velocity is

\[
Y_2(t) = -A \omega_n^2 \sin \omega_n t \\
= A \omega_n^2 \sin(\omega_n t + \pi)
\]

\[
Y_2(t) = -\omega_n^2 y(t)
\]

Hence \( Y_2(t) = -\omega_n^2 y(t) \)

Acceleration is proportional to the displacement

\[
Y_2(t) + \omega_n^2 y(t) = 0
\]

But

\[
\omega_n^2 = \frac{k}{m}
\]

\[
Y_2(t) + \frac{k}{m} y(t) = 0
\]

\[
Y_2(t) m + k y(t) = 0
\]

\[
m \ y_2(t) = \text{inertia force}
\]

\[
k \ y(t) = \text{Spring force}
\]
1. Determine the natural frequency and time period of the system as shown in fig. Take
\( E = 2.1 \times 10^5 \text{ N/mm}^2 \) and \( I = 13 \times 10^6 \text{ mm}^4 \)

To find the natural frequency

\[
\begin{align*}
 f &= \frac{1}{T} \\
 T &= \frac{2\pi}{\omega_n} \\
 &= 2\pi \sqrt{\frac{m}{k}} \\
 m &= \frac{w}{g} = \frac{500}{9.81 \times 1000} = 0.05096 \\
 T &= 2\pi \sqrt{\frac{500}{9.81 \times 1000} \times \frac{1}{120 \times 10^3}} \\
 T &= 0.004 \text{ sec}
\end{align*}
\]